Output, Emissions, and Technology: Some Thoughts

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Abstract: The principal goal of this article is to identify the implications of a binding emission constraint on a firm's optimal capital-labor ratio and to determine whether it is appropriate to write a firm's production function as an increasing function of its emissions alone. I find that even though a firm's supply curve may be written as a positive function of its emissions, it is not appropriate to write the production technology as an increasing function of only its emissions, except under special circumstances.

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1. Motivation

Environmental economists sometimes assume that a firm's output can be modeled as a positive function of emissions or, equivalently, a negative function of abatement. (For a couple of recent examples, see Shibayama and Fraser 2012 and Hallegatte et al. 2011.) The primary justification for this assumption is provided by Cropper and Oates (1992) who argue that treating emissions as an input is reasonable since reducing emissions diverts resources from other productive inputs. Copeland and Taylor (1994) prove the validity of this assumption under a very particular set of functional assumptions and it has been subsequently adopted by others (see, for example, Stokey 1998). Ebert and Welsch (2007) provide an alternative justification for this assumption using the materials balance equation as the main motivator.

Many of these models assume that output is a function of emissions and some other composite input. In essence, this means that the relative price of all non-emission inputs is held constant. In fact, the relative shadow prices of non-emission inputs generally change under the emission constraint. I analyze the implications of a binding emission constraint on firm's optimal input choices and ask whether there is a general economic basis for modeling output as a function of emissions alone.

2. The firm's problem and the optimal input ratio

For ease of exposition, I assume that output is a well behaved function of two inputs, capital (K) and labor (L), so that I obtain interior solutions throughout. I also assume perfectly competitive input and output markets. A firm's emissions are determined by its input use, and I assume that capital is the relatively more emission intensive input compared to labor, that is, I assume $e_K > e_L > 0$, where e_i ($i = K, L$) is the emission per unit of input.

2.1 With unconstrained emissions

In the absence of an emission constraint, the firm faces the following problem,

(1)   Minimize $TC_1 = wL + rK$

       subject to $f(L, K) = \bar{Q}$.

The Langrangian is $\mathcal{L}_1 = wL + rK + \lambda_1[\bar{Q} - f(L, K)]$, and the first order conditions for cost minimization are:

(2) $\frac{\partial \mathcal{L}_1}{\partial L} = w - \lambda_1 f_L = 0,$

(3) $\frac{\partial \mathcal{L}_1}{\partial K} = r - \lambda_1 f_K = 0,$ and

(4) $\frac{\partial \mathcal{L}_1}{\partial \lambda_1} = \bar{Q} - f(L, K) = 0.$

so that we get the familiar equilibrium condition that the marginal rate of technical substitution is equal to the ratio of input prices:

(5) $\frac{f_L}{f_K} = \frac{w}{r}.$

The solution to the first order conditions gives us the optimal capital-labor ratio, which I denote by K_1/L_1, as well as the optimal emission output, $e_1 = e_K K_1 + e_L L_1 > 0$. The Lagrange multiplier, $\lambda_1 > 0$ is the marginal cost of production. The total cost is $TC_1 = wL_1 + rK_1$.

2.2 In the presence of an emission constraint

Under a (binding) constraint on emissions, the firm's problem becomes:[1]

(6)   Minimize $TC_2 = wL + rK$

        subject to $f(L, K) = \bar{Q}$, and

        subject to $e_K K + e_L L = \bar{e} < e_1.$

The Lagrangian is $\mathcal{L}_2 = wL + rK + \lambda_2[\bar{Q} - f(L, K)] + \mu[\bar{e} - e_K K - e_L L]$, and we get the following first order conditions,

(7) $\frac{\partial \mathcal{L}_2}{\partial L} = w - \lambda_2 f_L - \mu e_L = 0,$

(8) $\frac{\partial \mathcal{L}_2}{\partial K} = r - \lambda_2 f_K - \mu e_K = 0,$

(9) $\frac{\partial \mathcal{L}_2}{\partial \lambda_2} = f(L, K) = \bar{Q},$ and

(10) $\frac{\partial \mathcal{L}_2}{\partial \mu} = \bar{e} - e_L L - e_K K = 0.$

The equilibrium condition is

(11) $\frac{f_L}{f_K} = \frac{w - \mu e_L}{r - \mu e_K},$

from which the emission-constrained optimal capital-labor ratio, K_2/L_2, is obtained. Note that the Lagrange multiplier on the emission constraint, μ, is the marginal cost of emissions so that in equilibrium $\mu < 0$: relaxing the emission constraint at the margin lowers the total cost of

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[1] For a similar problem with a non-linear emissions constraint see Blandford et al. (2011).

production. Thus, under a binding emission constraint, the effective prices of labor and capital are higher compared to their respective market prices. The total cost is $TC_2 = wL_2 + rK_2$.

Comparing equations (5) and (11), we cannot say *a priori* whether the optimal capital-labor ratio is higher, lower, or even the same under the emission constraint compared to the unconstrained case. Consider the following three cases:

Case 1: $K_1/L_1 = K_2/L_2$

Under this case, the input price ratio must be the same with and without the emission constraint so that, in equilibrium, the firm employs the same capital-labor ratio. That is, the right hand sides of equations (5) and (11) must be equal to each other, or

(12) $\dfrac{w}{r} = \dfrac{w - \mu e_L}{r - \mu e_K} \Rightarrow we_K = re_L \Rightarrow \dfrac{w}{r} = \dfrac{e_L}{e_K}.$.

In this case, the effective price of labor per unit of emissions is equal to the effective price of capital per unit of emissions, or the relative emission intensity of labor is the same as its relative price ratio. Therefore, in equilibrium the firm uses the same capital-labor ratio under the emission constraint as without it.

Case 2: $K_1/L_1 > K_2/L_2$

In this case, the firm employs a higher capital-labor ratio in the absence of the emission constraint than in the presence of it. This means that capital must be relatively cheaper in the absence of the emission constraint and that the relative price of labor effectively falls under the emission constraint. That is,

(13) $\dfrac{w}{r} > \dfrac{w - \mu e_L}{r - \mu e_K} \Rightarrow -\mu we_K > -\mu re_L \Rightarrow we_K > re_L \Rightarrow \dfrac{w}{r} > \dfrac{e_L}{e_K}.$

Under the emission constraint, even though the price per unit of emission is higher for labor than for capital, because the emission intensity of labor is less than its relative input price, the firm increases the amount of labor used relative to capital so that the equilibrium capital-labor ratio is lower under the emission constraint.

Case 3: $K_1/L_1 < K_2/L_2$

In this case, the firm employs a lower capital-labor ratio in the absence of the emission constraint than in the presence of it. This means that labor must be relatively cheaper in the absence of the emission constraint and that the relative price of labor effectively rises under the emission constraint making capital relatively cheaper. That is,

$$(1)\ \frac{w}{r} < \frac{w - \mu e_L}{r - \mu e_K} \Longrightarrow -\mu w e_K < -\mu r e_L \Longrightarrow w e_K < r e_L \Longrightarrow \frac{w}{r} < \frac{e_L}{e_K}.$$

Under the emission constraint, the price per unit of emission is lower for labor than for capital, but because the emission intensity of labor is greater than its relative input price, the firm reduces the amount of labor used relative to capital so that the equilibrium capital-labor ratio is higher under the emission constraint.

These three cases can be illustrated in the following diagrams. Solid lines refer to the situation in the absence of the emission constraint whereas the dashed lines refer to the situation under the emission constraint.

< Insert Figure 1 here>

3. Output and emissions

In each of these three cases, the emission constraint raises the effective price of both labor and capital. Thus, the total cost of producing a given quantity of output, $\bar{Q}$, rises, regardless of how the optimal capital-labor ratio changes. That is, in each case $TC_2(\bar{Q}) > TC_1(\bar{Q})$. Therefore, given output price, the profit maximizing firm will produce a lower quantity under the emission constraint that in its absence. In this (profit maximizing) sense, the firm's output is a positive function of emissions or a negative function of abatement. One can, indeed, write the firm's supply curve as a positive function of emissions.

Is it possible to write the firm's output as a positive function of its emission using the technology relationship alone and without invoking profit maximization (or the materials balance)? Consider the following diagram in which I show the firm production technology through its isoquant, and the resulting emission level, represented by the dashed emission budget line.

< Insert Figure 2 here>

Figure 2 shows an isoquant for some arbitrary level of output, $Q_1 > 0$. Point A lies on the isoquant and corresponds to K_A and L_A amounts of capital and labor, respectively. Point B lies above the isoquant and therefore corresponds to a higher level of output that is produced using K_B and L_B amounts of the two inputs, respectively.

The diagram also shows two different emission production technologies, e^1 (heavy dashes) and e^2 (lighter dots). In each case, the absolute value of the slope of the emission budget is the relative emission intensity, e_L/e_k. (Note, in both cases, I assume that capital is the relatively emission intensive input.) What is different between the two emission technologies is the slope relative to the slope of the production isoquant at point A.

In the case of technology e^1 it is clear that $e_B^1 > e_A^1$, that is, emissions are higher at point B compared to A. This is because the (absolute value of the) slope of the emission budget e^1 is greater than the slope of the isoquant at A. However, in the case of technology e^2 the slope of the emission budget is less than $MRTS_{K,L}$ at A and we have that $e_B^1 < e_A^1$. This shows that depending on the degrees of substitutability of capital and labor in the production technology for output relative to the degree of substitutability of these inputs in the production of emissions, it may be technologically possible to produce a higher (or the same) level of output with lower emissions.

4. Conclusion

The discussion in sections 2 and 3 implies that even with a well behaved production function, it is generally not possible to rewrite a standard two-input production function as an increasing function of emissions even when the relative market input prices are constant. Authors should be cognizant of the implicit assumptions regarding the production technology and input prices when modeling output as a positive function of emissions or negative function of abatement. However, it is true that, given input prices, a profit maximizing firm will always produce a lower output under a binding constraint on emission than without, and there are no special assumptions required for this to hold.

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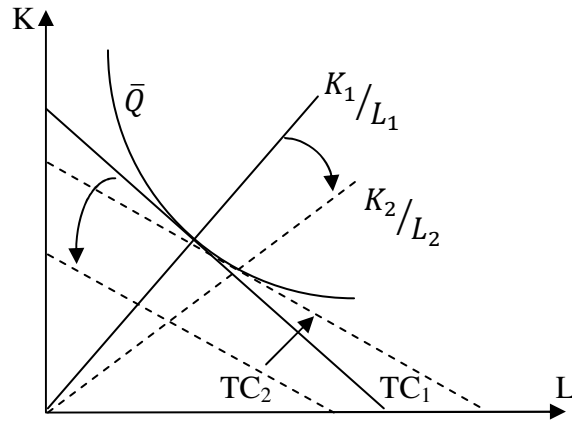
Figure 1: Optimal capital-labor ratio with and without an emission constraint

Case 1: Optimal capital-labor ratio is unchanged under the emission constraint
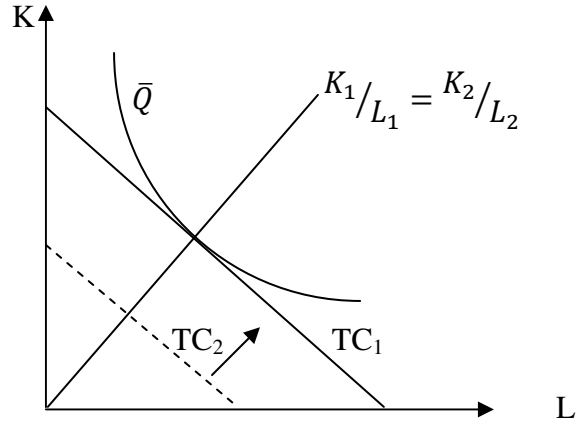
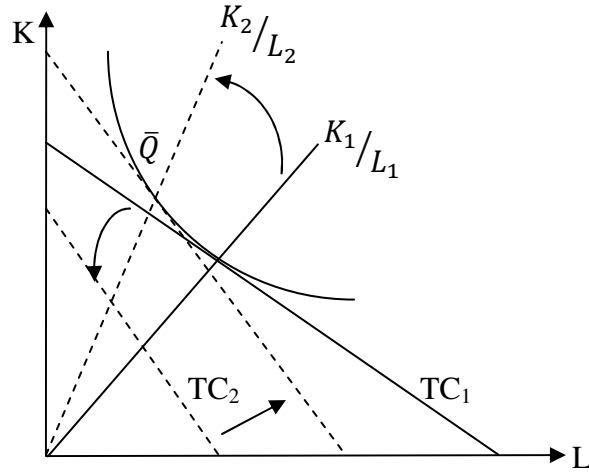$$|Slope\ TC_1| = \frac{w}{r} = \frac{w - \mu e_L}{r - \mu e_K} = |Slope\ TC_2|$$

Case 2: Optimal capital-labor ratio is lower under the emission constraint

$$|Slope\ TC_1| = \frac{w}{r} > \frac{w - \mu e_L}{r - \mu e_K} = |Slope\ TC_2|$$

Case 3: Optimal capital-labor ratio is higher under the emission constraint (Case 3)

$$|Slope \; TC_1| = \frac{w}{r} < \frac{w - \mu e_L}{r - \mu e_K} = |Slope \; TC_2|$$

Figure 2: Output and emissions

$Q_1 = f(K, L)$

e_B^1

e_A^2

e_A^1

e_B^2

K_A

A

B

K_B

L_A L_B

K

L