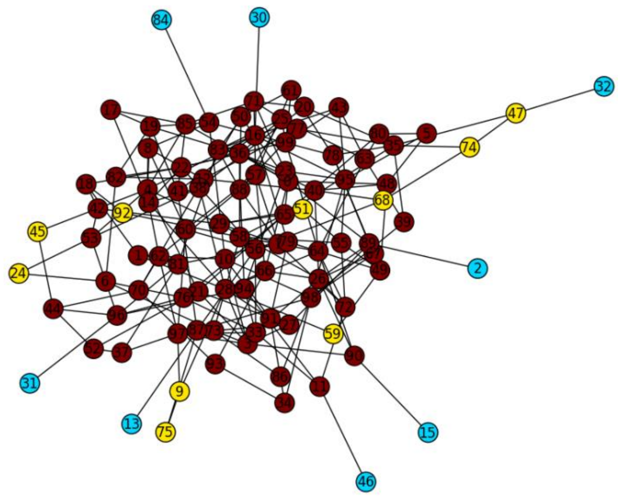
Topological Analysis (1)



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Network data import & export

- read_gml

- read_adjlist
- read_edgelist
  - Creates undirected graphs by default; use "create_using=NX.DiGraph()" option to generate directed graphs

Exercise

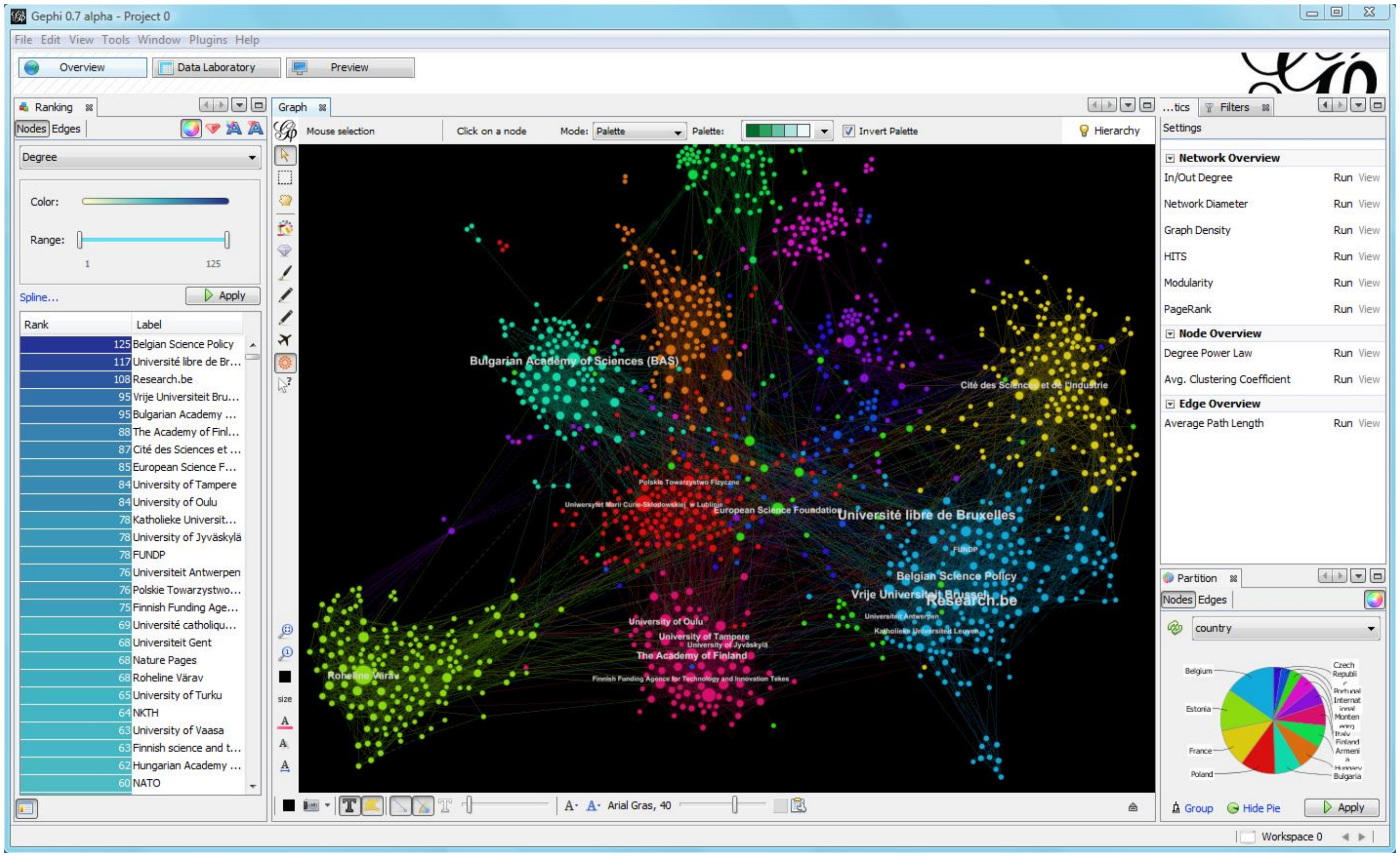
- Import Supreme Court Citation Network Data into NetworkX (http://jhfowler.ucsd.edu/judicial.htm)
  - Import as an undirected graph
  - Import as a directed graph

Network visualization

- “nx.draw”
- Various layout functions
  - Spring, circular, random, spectral, etc.
- For visualization of large-scale networks, use “Gephi”

Gephi

- Network visualization & analysis tool



Basic Properties of Networks

Basic properties of networks

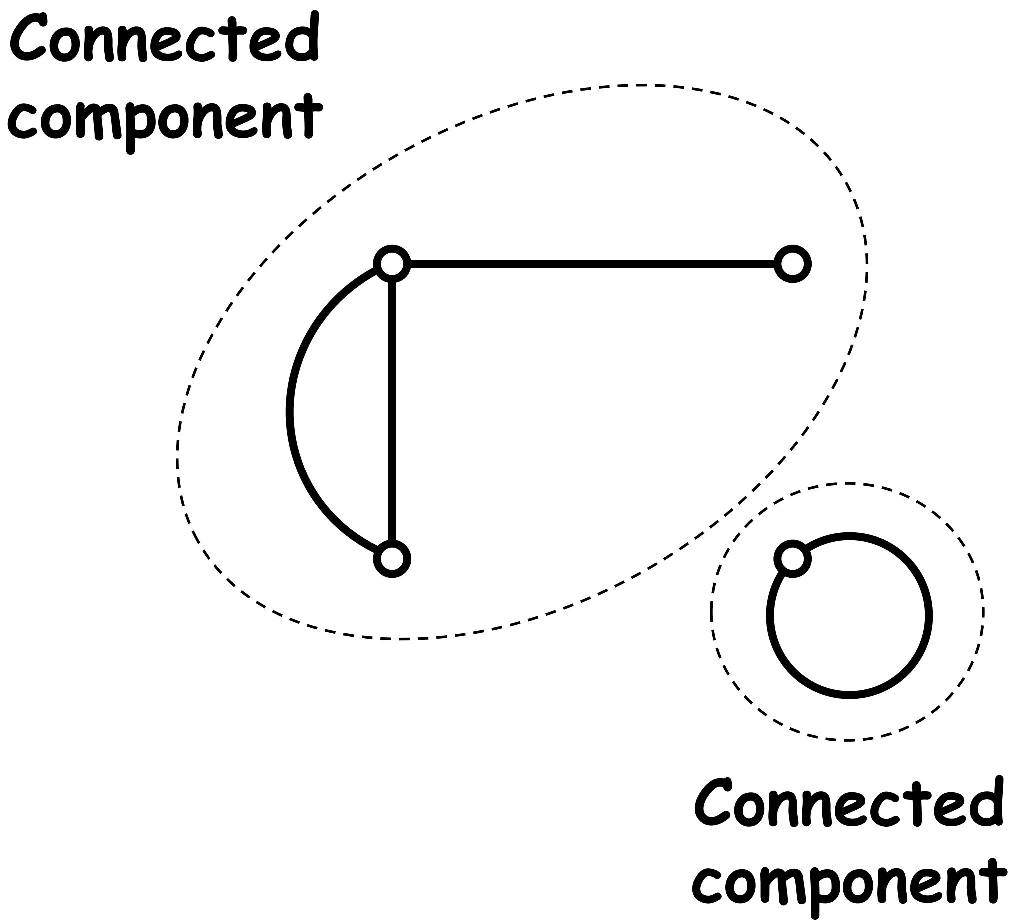
- Number of nodes
- Number of links
- Network density

- Connected components

Network density

- The ratio of # of actual links and # of possible links
  - For an undirected graph:
    $$d = |E| / (|V| (|V| - 1) / 2)$$
  - For a directed graph:
    $$d = |E| / (|V| (|V| - 1))$$

Connected components



Number of connected components = 2

Exercise

- Measure the following for the (undirected) Supreme Court Citation Network
  - Number of nodes, links
  - Network density
  - Number of connected components
  - Size of the largest connected component
  - Distribution of the sizes of connected components

Shortest path lengths, etc.

- shortest_path
- shortest_path_length
- eccentricity
  - Max shortest path length from each node
- diameter
  - Max eccentricity in the network
- radius
  - Min eccentricity in the network

Exercise

- Draw the Karate Club network with its nodes painted with different colors according to their eccentricity

Characteristic path length

- Average shortest path length over all pairs of nodes
- Characterizes how large the world represented by the network is
  - A small length implies that the network is well connected globally

Clustering coefficient

- For each node:
  - Let n be the number of its neighbor nodes
  - Let m be the number of links among the k neighbors
  - Calculate c = m / (n choose 2)

  Then C = <c> (the average of c)
- C indicates the average probability for two of one's friends to be friends too
  - A large C implies that the network is well connected locally to form a cluster

Exercise

- Measure the average clustering coefficients of the following network:
  - Karate Club graph
  - Krackhardt Kite graph
  - Supreme Court Citation network
  - Any other network of your choice
- Compare them and discuss
  - Can you tell anything meaningful?

Centralities

Centrality measures ("B,C,D,E")

- Degree centrality
  - How many connections the node has
- Betweenness centrality
  - How many shortest paths go through the node
- Closeness centrality
  - How close the node is to other nodes
- Eigenvector centrality

Degree centrality

- Simply, # of links attached to a node

$$C_D(v) = deg(v)$$

or sometimes defined as

$$C_D(v) = deg(v) / (N-1)$$

Betweenness centrality

- Prob. for a node to be on shortest paths between two other nodes

$$C_B(v) = \frac{1}{(n-1)(n-2)} \sum_{s \neq v, e \neq v} \frac{\#sp_{(s,e,v)}}{\#sp_{(s,e)}}$$

- s: start node, e: end node
- $\#sp_{(s,e,v)}$: # of shortest paths from s to e that go though node v
- $\#sp_{(s,e)}$: total # of shortest paths from s to e
- Easily generalizable to "group betweenness"

Closeness centrality

- Inverse of an average distance from a node to all the other nodes

$$C_C(v) = \frac{n-1}{\sum_{w \neq v} d(v,w)}$$

- d(v,w): length of the shortest path from v to w
- Its inverse is called "farness"
- Sometimes "Σ" is moved out of the fraction (it works for networks that are not strongly connected)
- NetworkX calculates closeness within each connected component

Eigenvector centrality

- Eigenvector of the largest eigenvalue of the adjacency matrix of a network

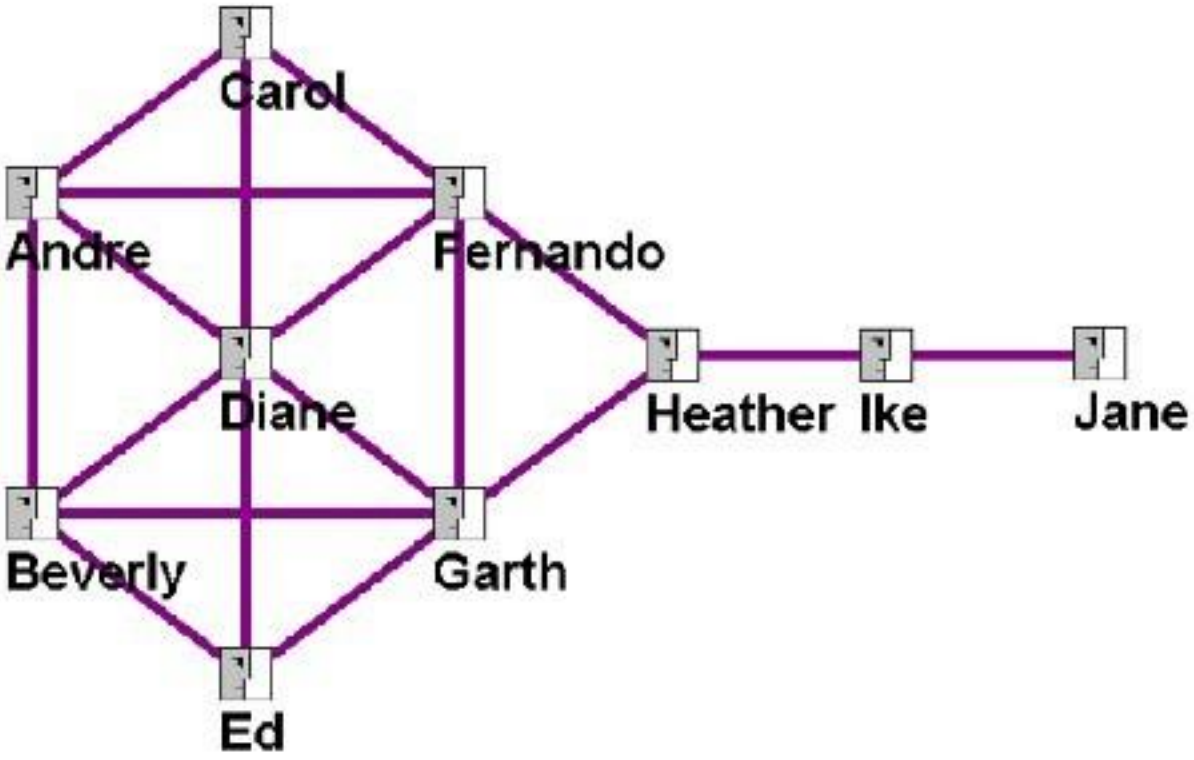
$$C_E(v) = (v\text{-th element of } x)$$

$$Ax = \lambda x$$

- λ: dominant eigenvalue
- x is often normalized ($|x| = 1$)

Exercise

- Who is most central by degree, betweenness, closeness, eigenvector?



Which centrality to use?

- To find the most popular person
- To find the most efficient person to collect information from the entire organization
- To find the most powerful person to control information flow within an organization
- To find the *most important* person (?)

Exercise

- **Measure four different centralities for all nodes in the Karate Club network and visualize the network by coloring nodes with their centralities**

Exercise

- Create a directed network of any kind and measure centralities
- Make it undirected and do the same
  - How are the centrality measures affected?

Randomizing Network Topologies

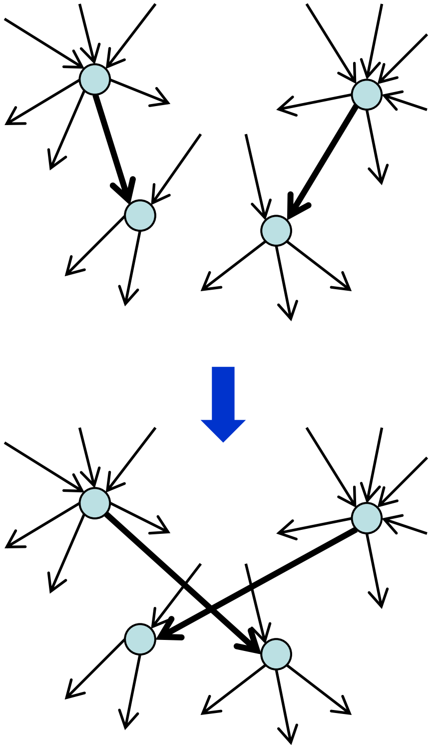
Randomizing networks

- Construct a "null model" network samples to test statistical significance of experimentally observed properties
  - Randomized while some network properties are preserved (e.g., degrees)
  - If the observed properties still remain after randomization, they were simply caused by the preserved properties
  - If not, something else was causing them

Randomlization method (1)

- Double edge swap method

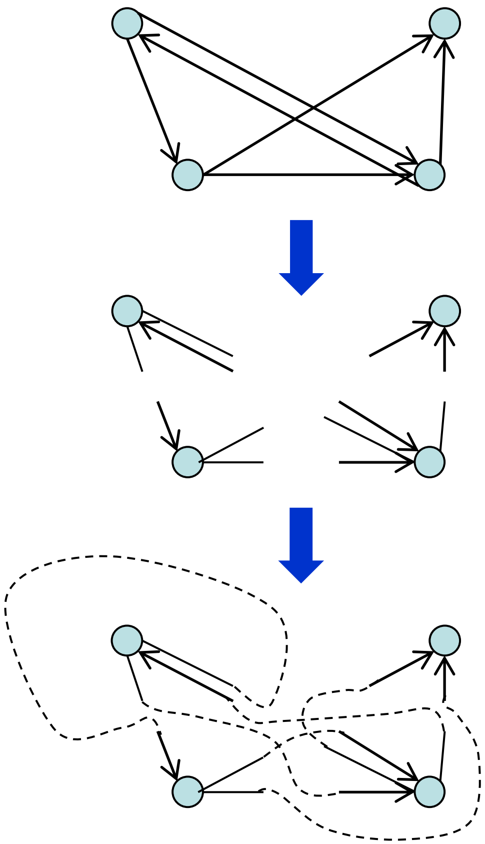
1. Randomly select two links
2. Swap its end nodes
   - (If this swap destroys some network property that should be conserved, cancel it)
3. Repeat above many times



Randomlization method (2)

- Configuration model (Newman 2003)

1. Cut every link into halves (heads and tails)
2. Randomly connect head to tail
   - This conserves degree sequences
   - (Could result in multiple links and self-loops)



Other randomization methods

- Keeping only #'s of nodes and edges
- Degree sequence method
- Expected degree sequence method

Exercise

- Randomize connections in the Karate Club graph
- Measure the average clustering coefficient of the randomized network many times
- Test whether the average clustering coefficient of the original network is significantly non-random or not