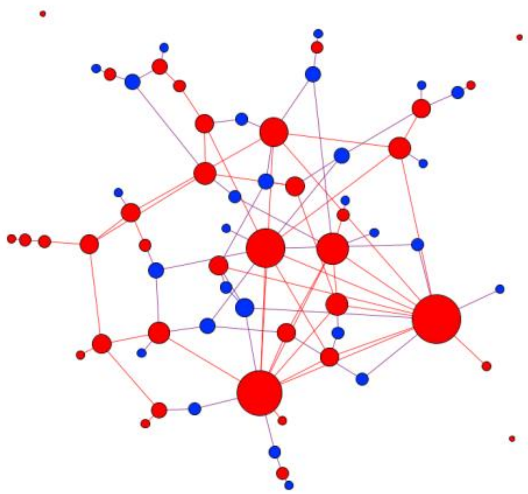
Simulation III: Adaptive Networks



Hiroki Sayama
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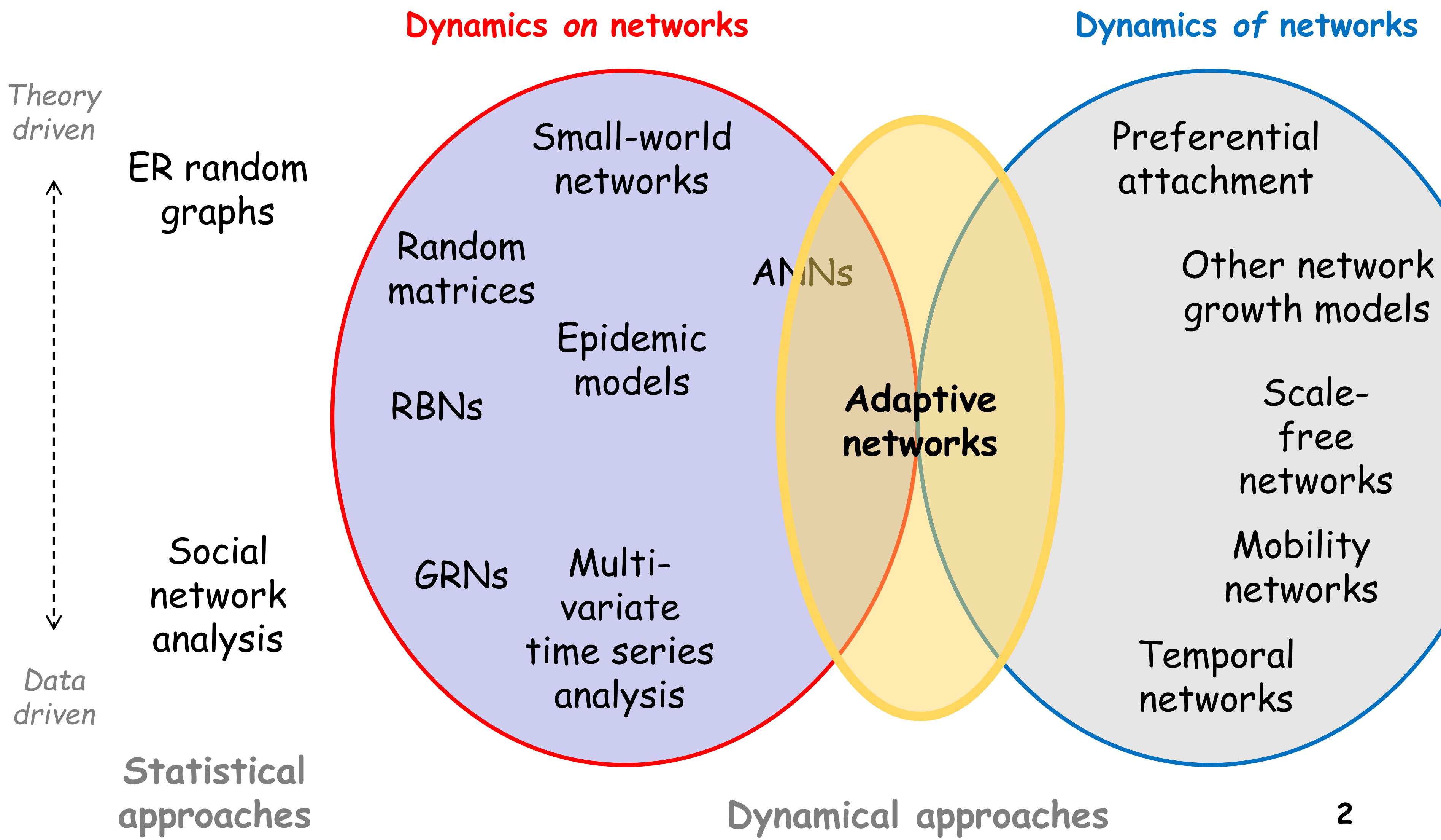
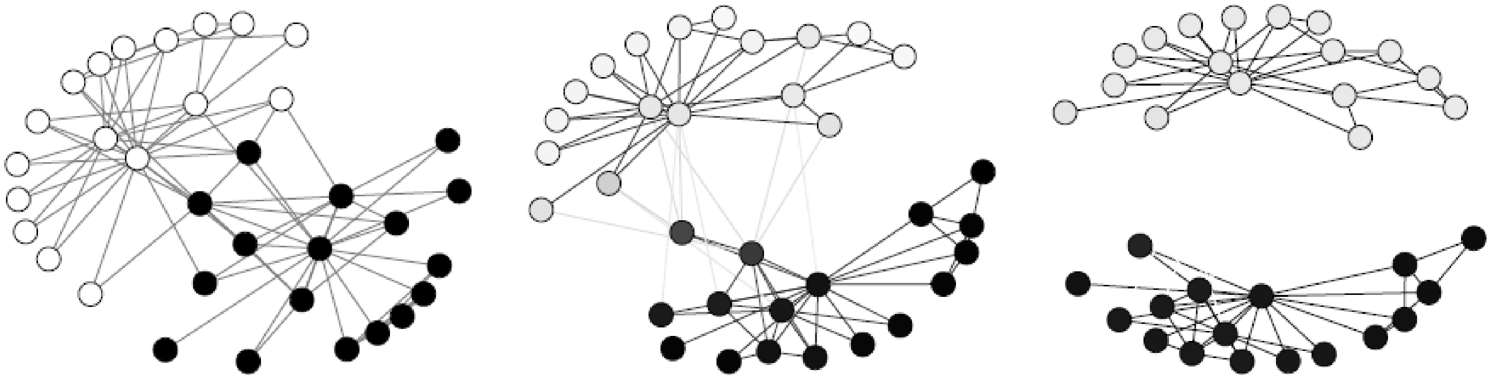
A map of network science



Adaptive networks

- Complex networks whose states and topologies co-evolve, often over similar time scales
  - Link (node) states adaptively change according to node (link) states



Adaptive networks in action

- **Many real-world complex systems show coupling between "dynamics *of* networks" and "dynamics *on* networks"**

System	Nodes	Edges	States of nodes	Topological changes
Organism	Cells	Intercellular communication channels	Gene/protein activities	Fission and death of cells during development
Ecological community	Species	Interspecific relationships	Population	Speciation, invasion, extinction of species
Human society	Individual	Conversations, social relation-ships	Social, professional, economical, political, cultural statuses	Changes in social relationships, entry and withdrawal of individuals
Communica-tion network	Terminals, hubs	Cables, wireless connections	Information stored and transacted	Addition and removal of terminal or hub nodes

Simulation of Adaptive Networks

Simulating state-topology coevolution

- Technically, very easy; not so much different from other network simulation models

- One minor problem:
  How to handle topological changes while state changes are also ongoing?
  → Asynchronous updating

Example: Epidemics on adaptive networks

- Original epidemic network model

  + adaptive changes of links

- A susceptible node that has a link to an infected node will cut the link and reconnect it to another susceptible node with probability p_c
- Does the disease stay in the network?

Exercise

- Study the effects of rewiring probability on the disease fixation on and the global network structure of an initially random social network
  - In what condition will the disease remain within society?
  - How will the topology of the network be reformed through the disease propagation process?

Example: Adaptive voter model

- Original voter model
  - + adaptive disconnection of links
- A link that connects two nodes with different opinion states may be cut with probability p_c
- How will the social network and opinions evolve?

Exercise

- Study the effects of the link disconnection probability on the consensus formation in the adaptive voter model
  - Plot the final number of opinions as a function of p_c
  - How will the topology of the network be changed by the diversity of opinions?

Example: Adaptive diffusion model

- Original diffusion model

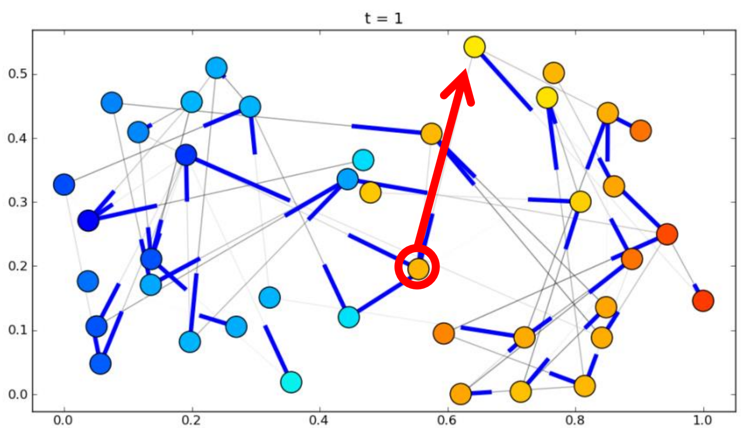
  + adaptive disconnection of links

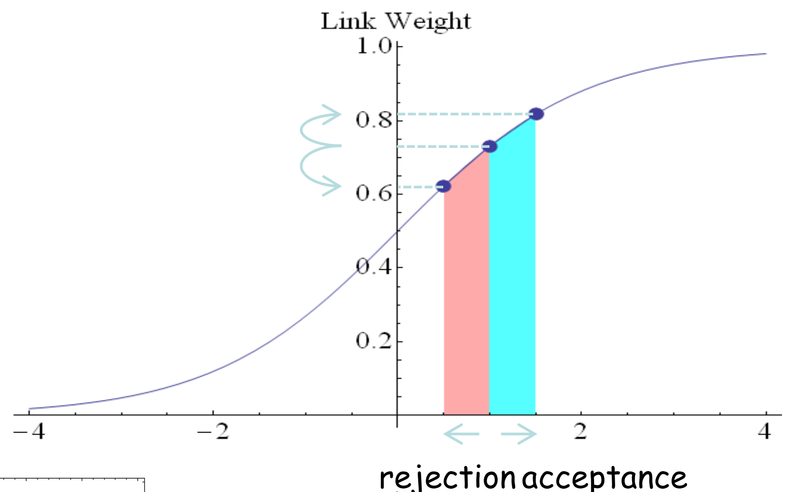
- Link weights will increase or decrease based on the similarity/dissimilarity of node states across the links

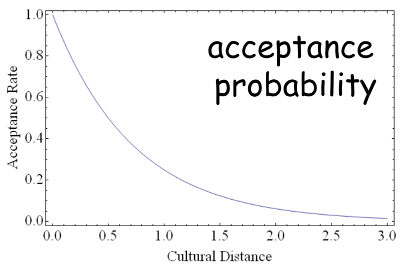
  - Conceptually similar to the adaptive voter model

Application 1: Corporate merger

- **Modeling and simulation of cultural integration in two merging firms**

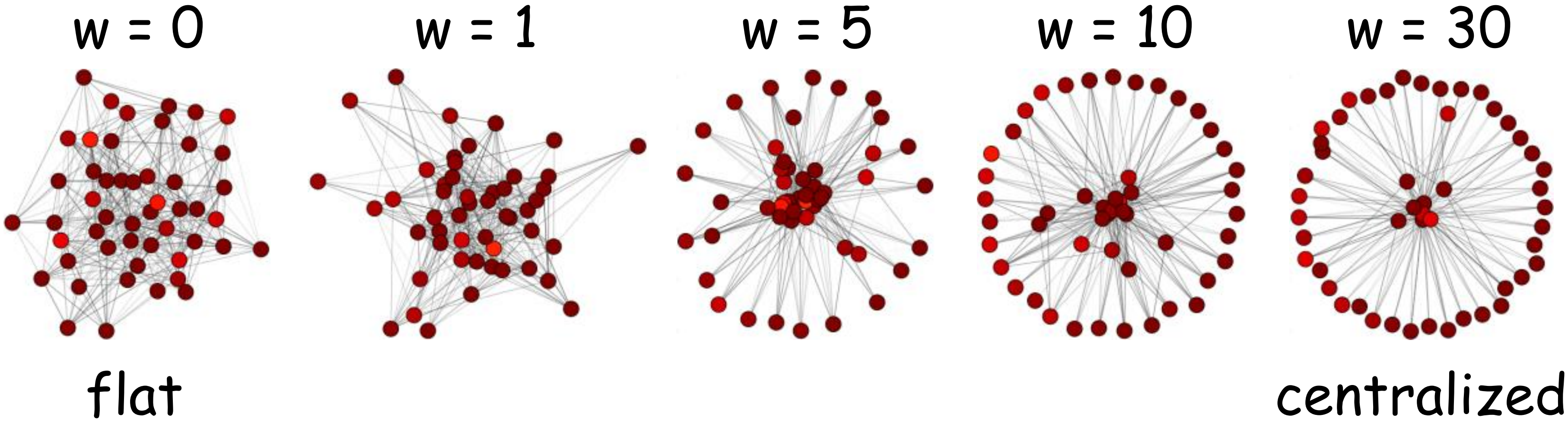






Yamanoi & Sayama, *Comput. Math. Org. Theory* 19, 516-537, 2013.

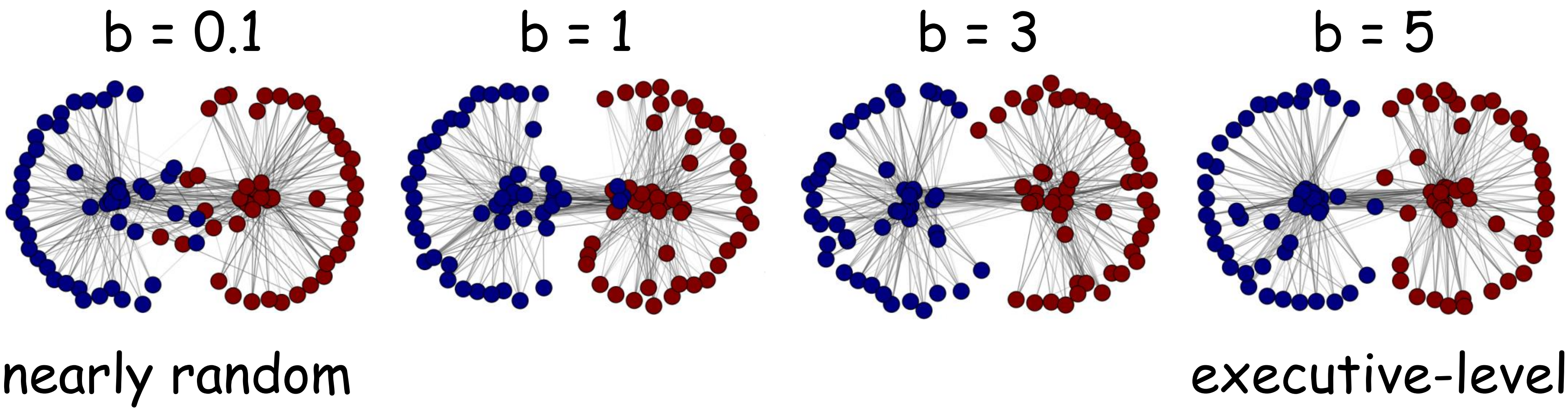
"Within-firm" concentration (w)



- **Prob. for node i to become an info source:**

$$P_w(i) \sim (i/n)^w \qquad (i = 1, 2, \ldots, n; \; n = \text{firm size})$$

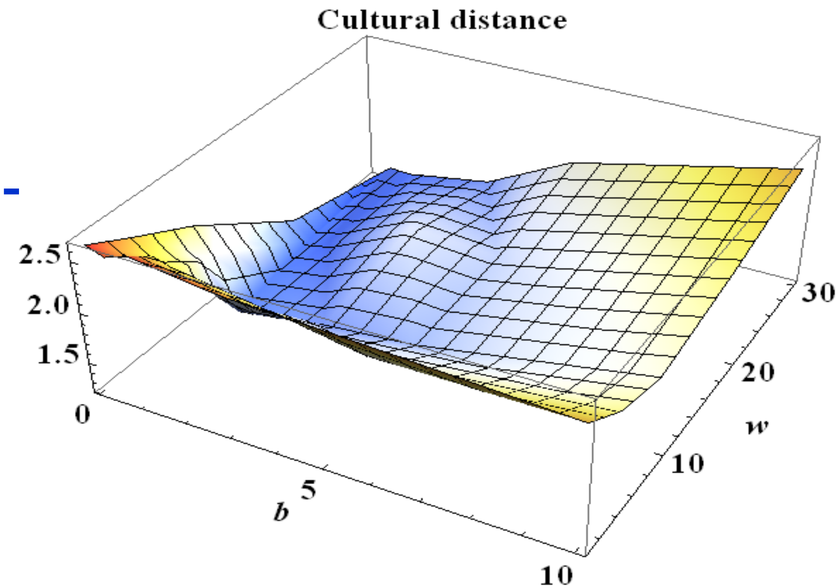
“Between-firm” concentration (b)

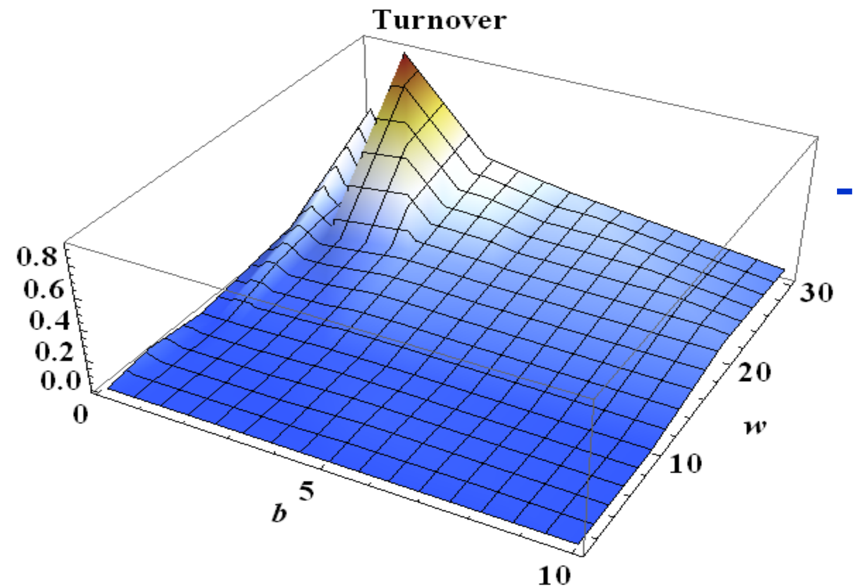


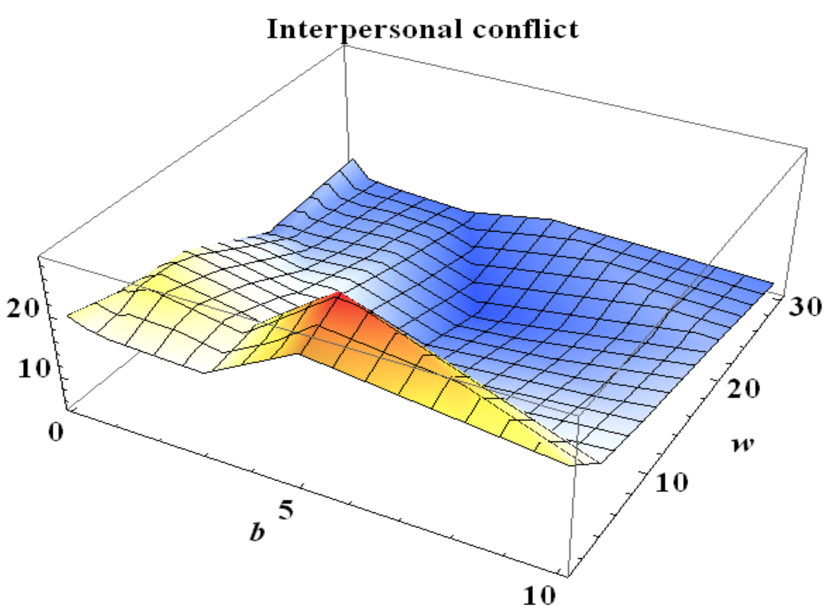
- Prob. for node i to have an inter-firm tie:

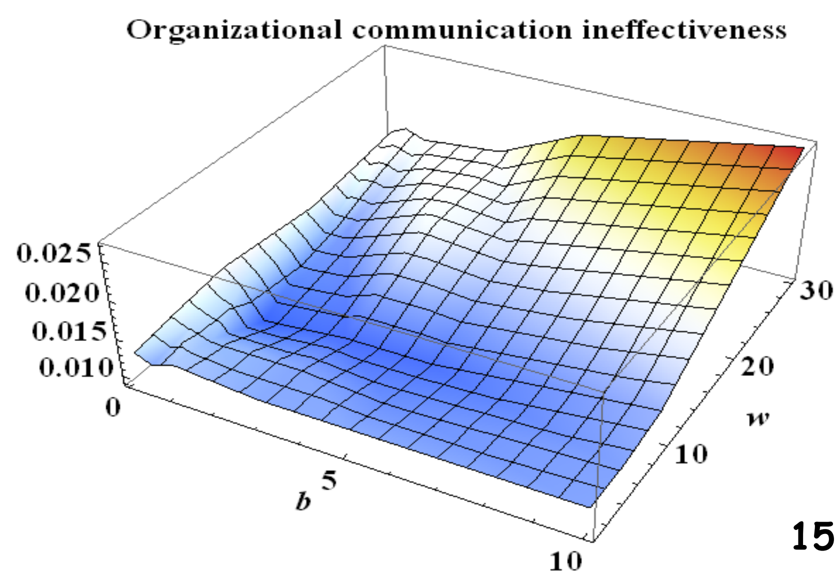
$P_b(i) \sim c_i{}^b$

(c_i = within-firm closeness centrality of i)


Cultural distance
2.5
2.0
1.5
0
5
b
10
10
20
30
w


Turnover
0.8
0.6
0.4
0.2
0.0
0
5
b
10
10
20
30
w


Interpersonal conflict
20
10
0
5
b
10
10
20
30
w


Organizational communication ineffectiveness
0.025
0.020
0.015
0.010
0
5
b
10
10
20
30
w

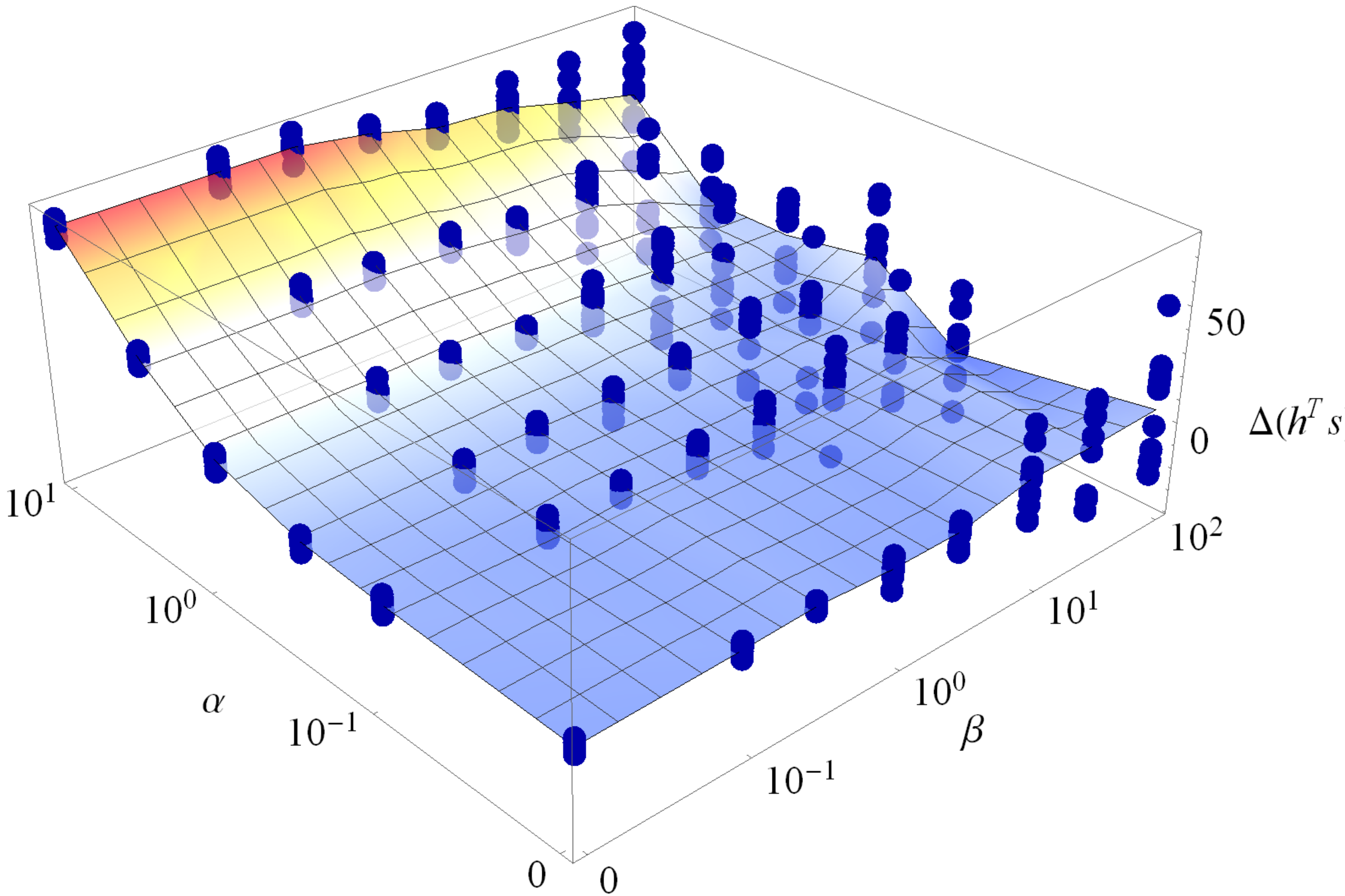
Application 2: Social diffusion and global drift

- Sayama & Sinatra, PRE 91, 032809, 2015

$$\frac{ds_i}{dt} = c\left(\langle s_j \rangle_j^i - s_i\right)$$

Adaptive link weight adjustment:

$$\frac{da_{ij}}{dt} = a_{ij}\left[\alpha \frac{s_i + s_j - 2\langle s \rangle}{2\sigma_s} - \beta \frac{(k_i - \langle k \rangle)(k_j - \langle k \rangle)}{\sigma_k^2}\right]$$



α
β
Δ(h^T s)
10^1
10^0
10^-1
0
10^2
10^1
10^0
10^-1
0
50
0

Exercise

- Change the rule of link weight adjustment in the adaptive diffusion model
  - E.g., Sayama & Sinatra (2015)

- Simulate the revised model and see how the network topology and state co-evolve

Theoretical Framework:

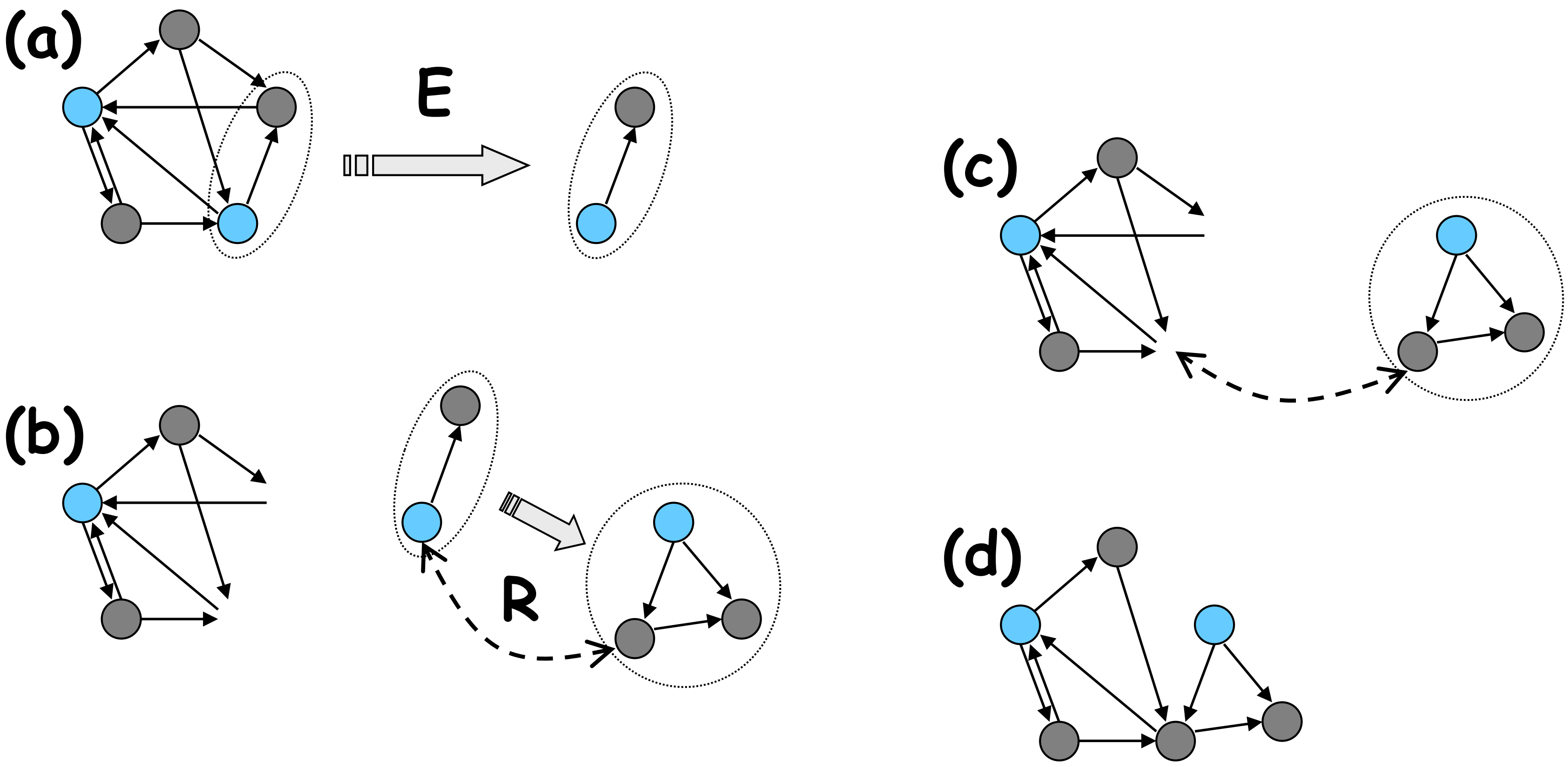
Generative Network Automata

Generative network automata

- Unified representation of dynamics *on* and *of* networks using graph rewriting
- Defined by <E, R, I>:
  - E : Extraction mechanism — When, Where
  - R : Replacement mechanism — What
  - I : Initial configuration

Sayama, *Proc. 1st IEEE Symp. Artif. Life*, 2007, pp.214-221.

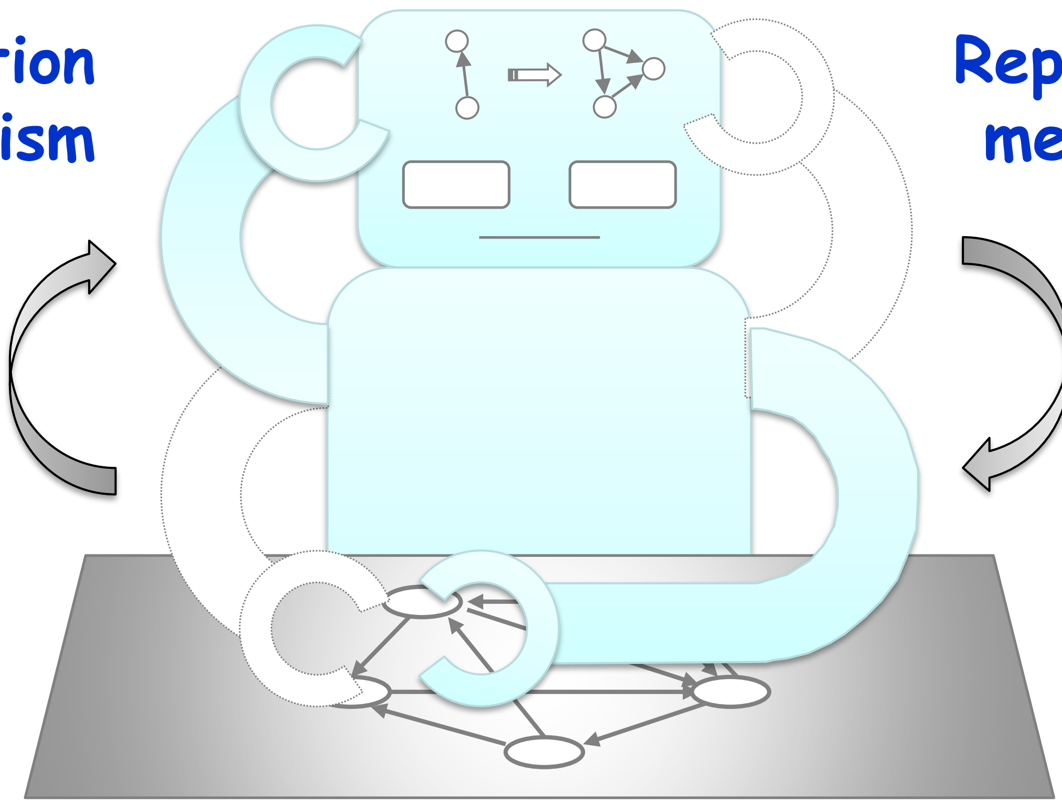
GNA rewriting example



Actually, it's a generative network automat~~a~~-on

E :
Extraction mechanism

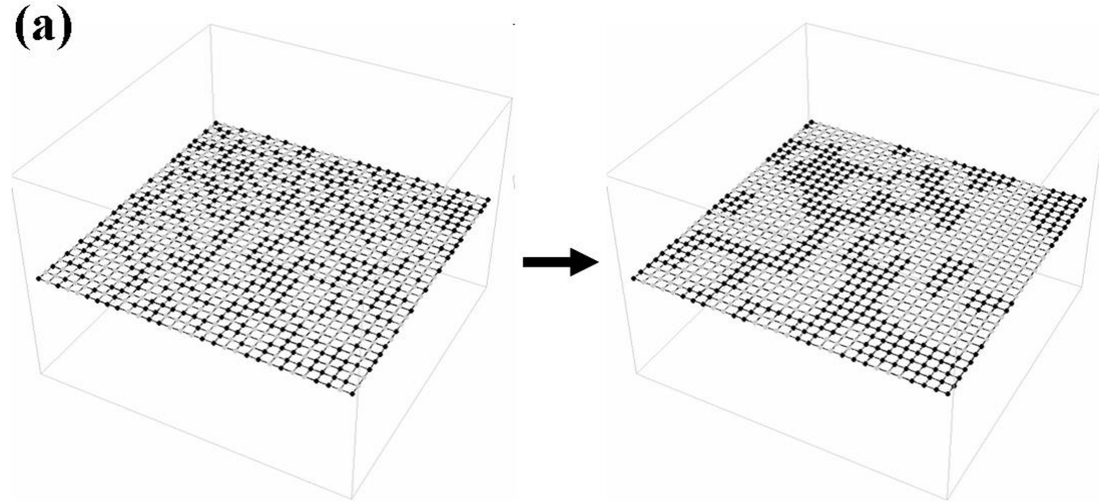
R:
Replacement mechanism



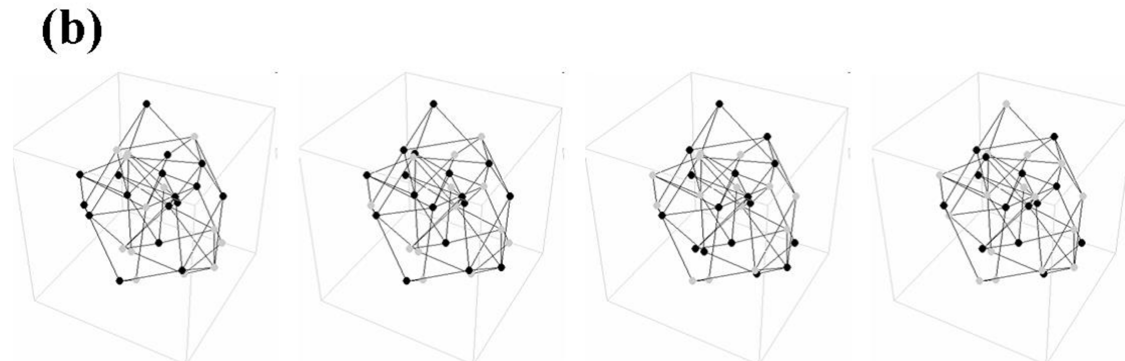
Generality of GNA

- GNA can uniformly represent in <E, R, I>:
  - Conventional dynamical systems models
    - If R always conserves local network topologies and modifies states of nodes only
    - E.g. cellular automata, Boolean networks
  - Complex network generation models
    - If R causes no change in local states of nodes and modifies topologies of networks only
    - E.g. small-world, scale-free networks

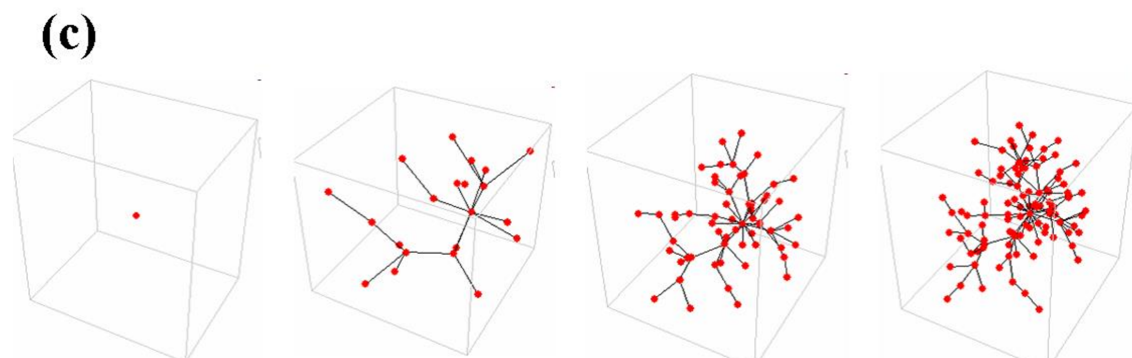
Cellular automata



Random Boolean network



BA scale-free network



Exhaustive search of rules

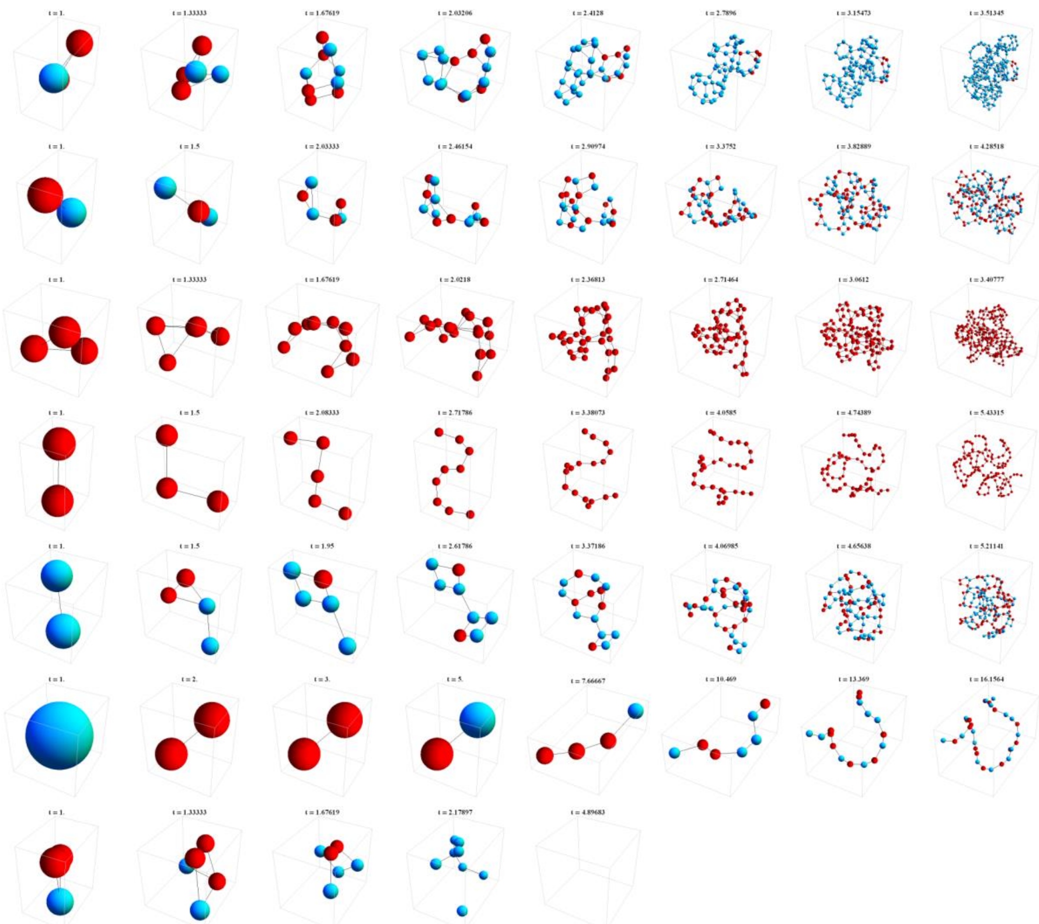
- E samples a node randomly and then extracts an induced subgraph around it
- R takes 2-bit inputs (states of the node and neighbors) and makes 1-out-of-10 decisions
  - Total number of possible R's: $10^{2^2} = 10{,}000$

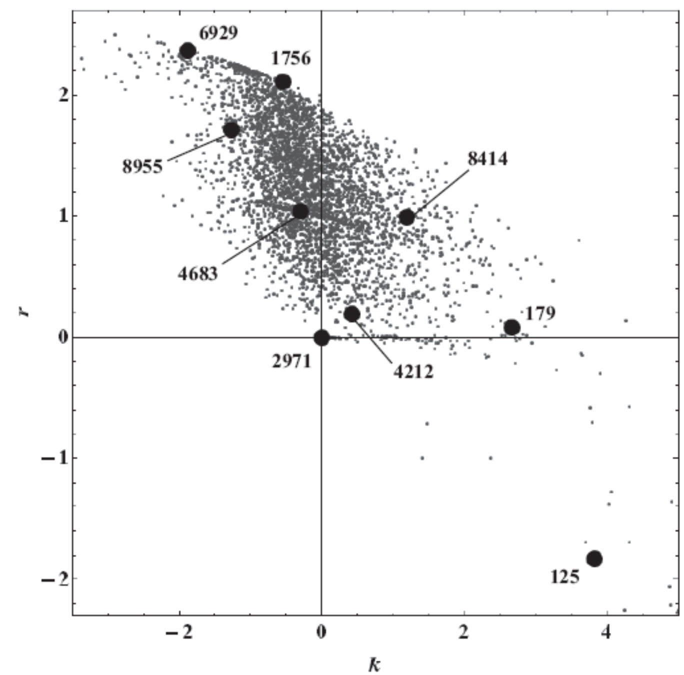
- "Rule Number" rn(R) is defined by

$$rn(R) = a_{11}\ 10^3 + a_{10}\ 10^2 + a_{01}\ 10^1 + a_{00}\ 10^0$$

  - $a_{ij} \in \{0, 1, \ldots 9\}$: Choices of R when state of u is i and local majority state is j

Exhaustive search of rules



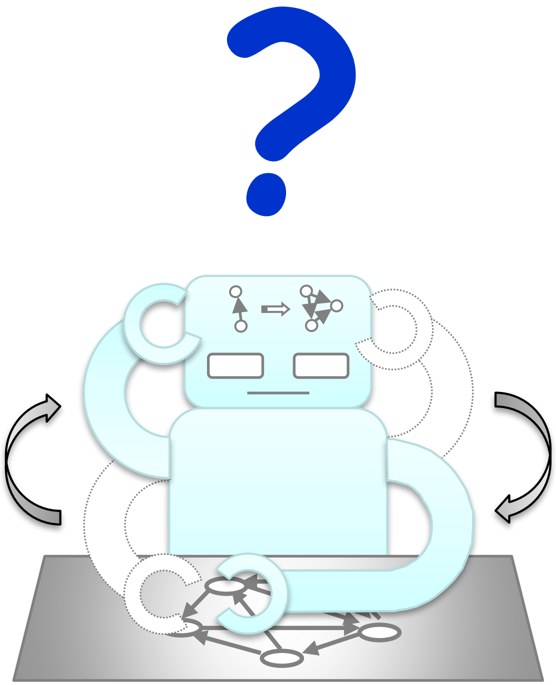


Sayama & Laramee, *Adaptive Networks*, Springer, 2009, pp.311-332.

Developing Adaptive Network Models from Empirical Data

A challenge

- How to derive a set of dynamical rules directly from empirical data of network evolution?
- Separation of extraction and rewriting in GNA helps the rule discovery



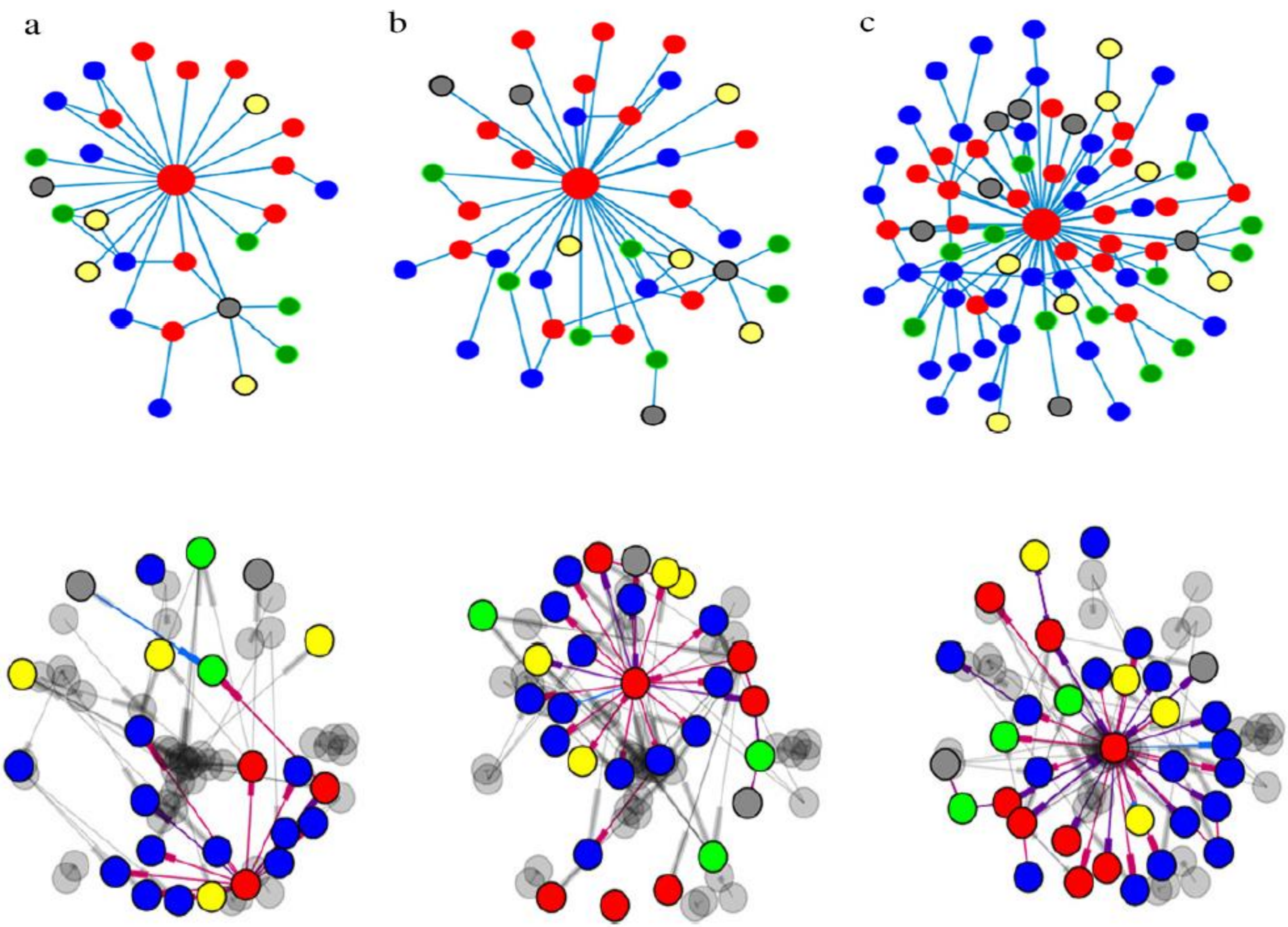
Pestov, Sayama, & Wong, *Proc. 9th Intl. Conf. Model. Simul. Visual. Methods*, 2012.

Schmidt & Sayama, *Proc. 4th IEEE Symp. Artif. Life*, 2013, pp.27-34.

Application to operational network modeling

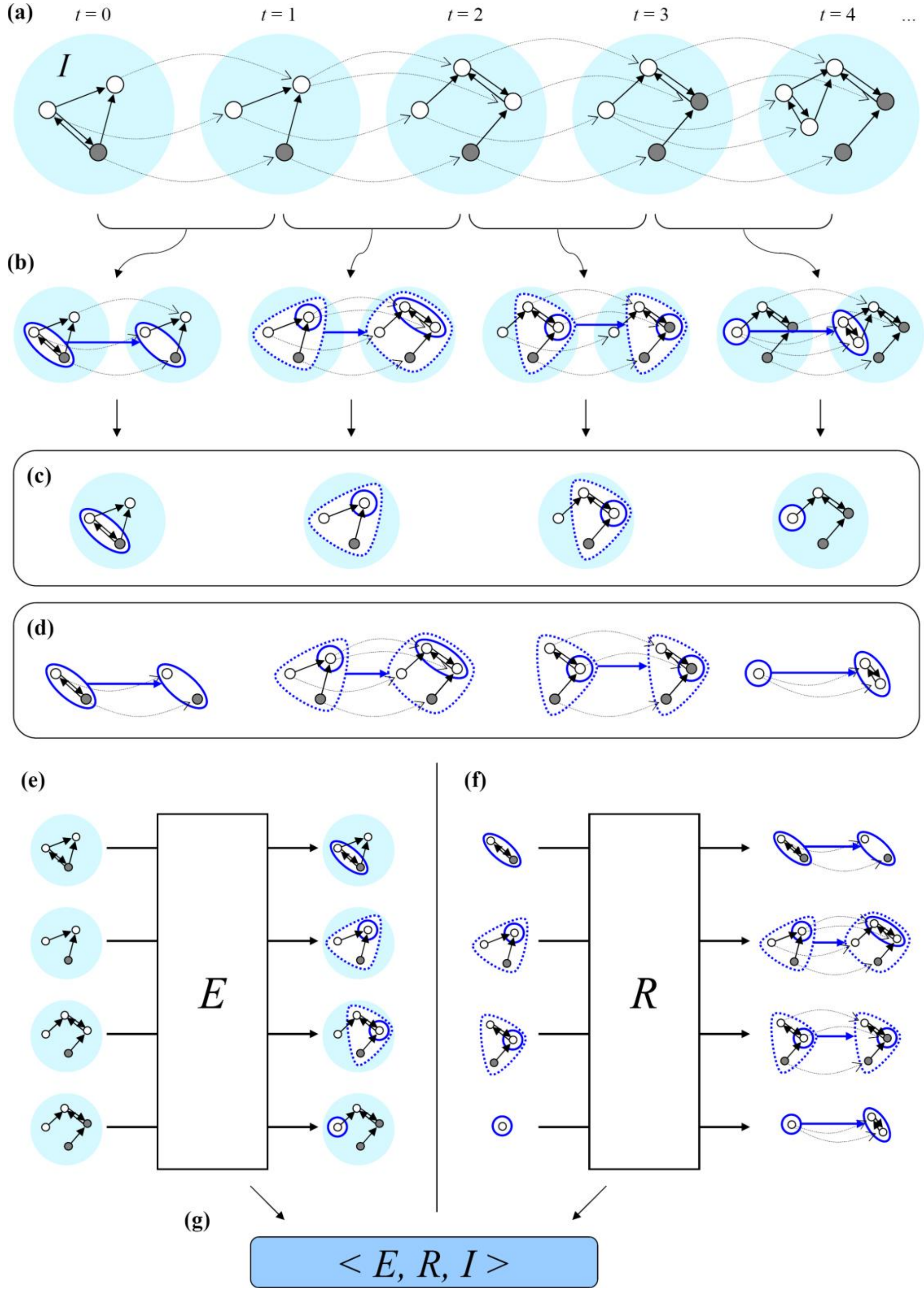
- Canadian Arctic SAR (Search And Rescue) operational network
  - Rewriting rules manually built directly from actual communication log of a December 2008 SAR incident

  - Developed a simulator for hypothetical SAR operational network development

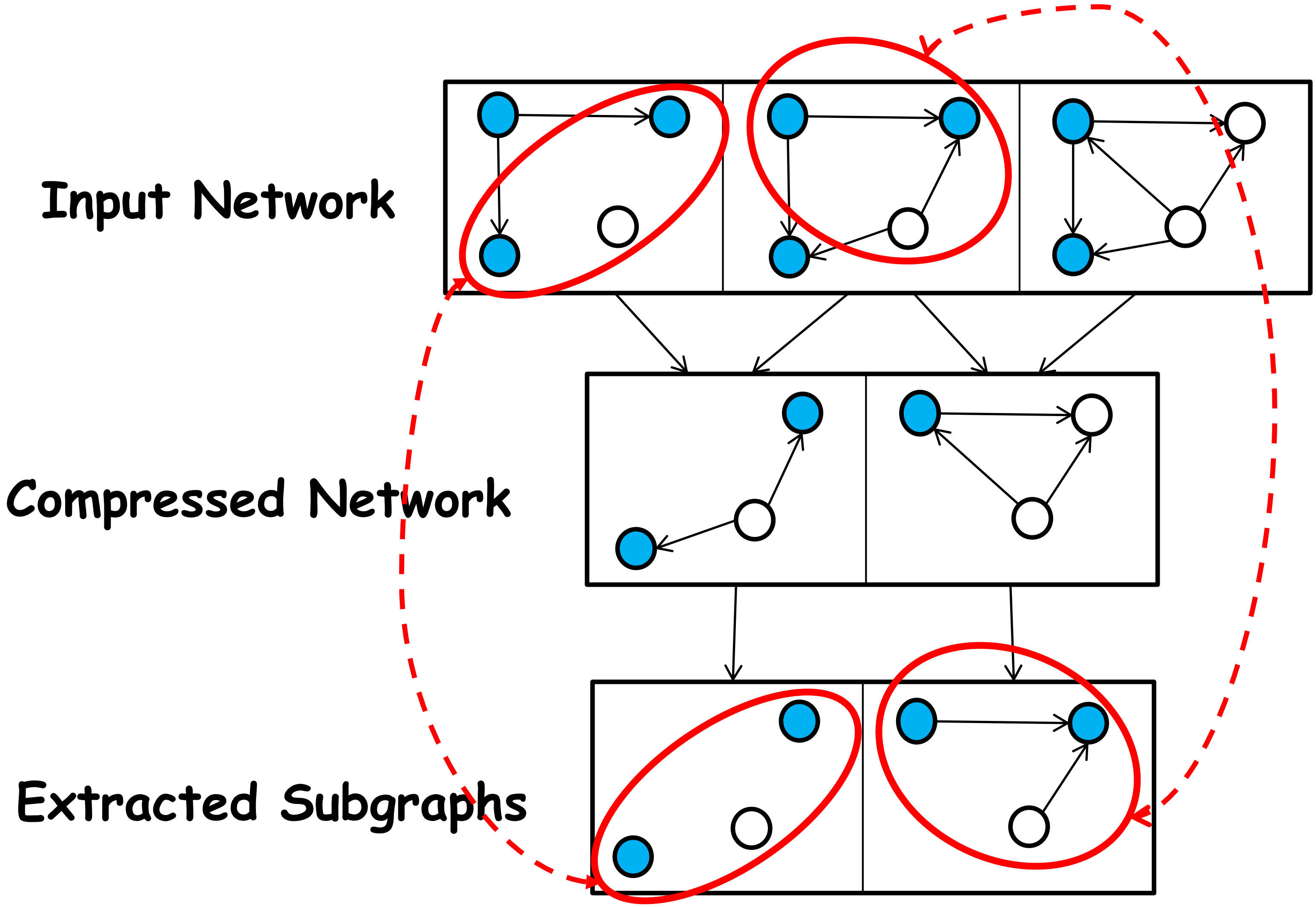

a
b
c

Automation of model discovery from data: PyGNA

- Adaptive network rule discovery and simulation implemented in Python
  - https://github.com/schmidtj/PyGNA
- Input: Time series of network snapshots
- Output: A GNA model that best describes given data


(a)
t = 0
t = 1
t = 2
t = 3
t = 4
...
I
(b)
(c)
(d)
(e)
E
(f)
R
(g)
< E, R, I >

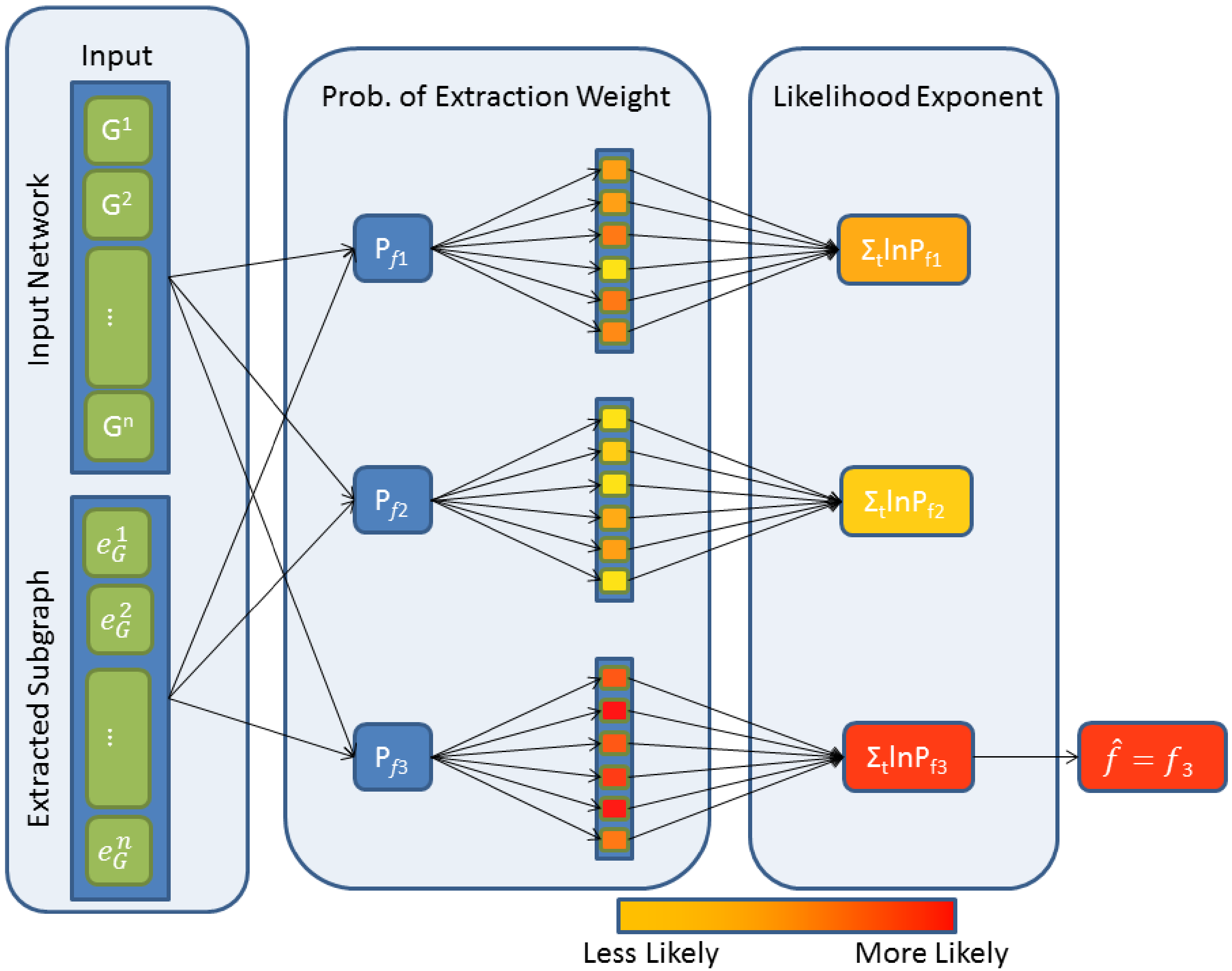
Extracted subgraphs



Extraction mechanism identification: “Where, when”

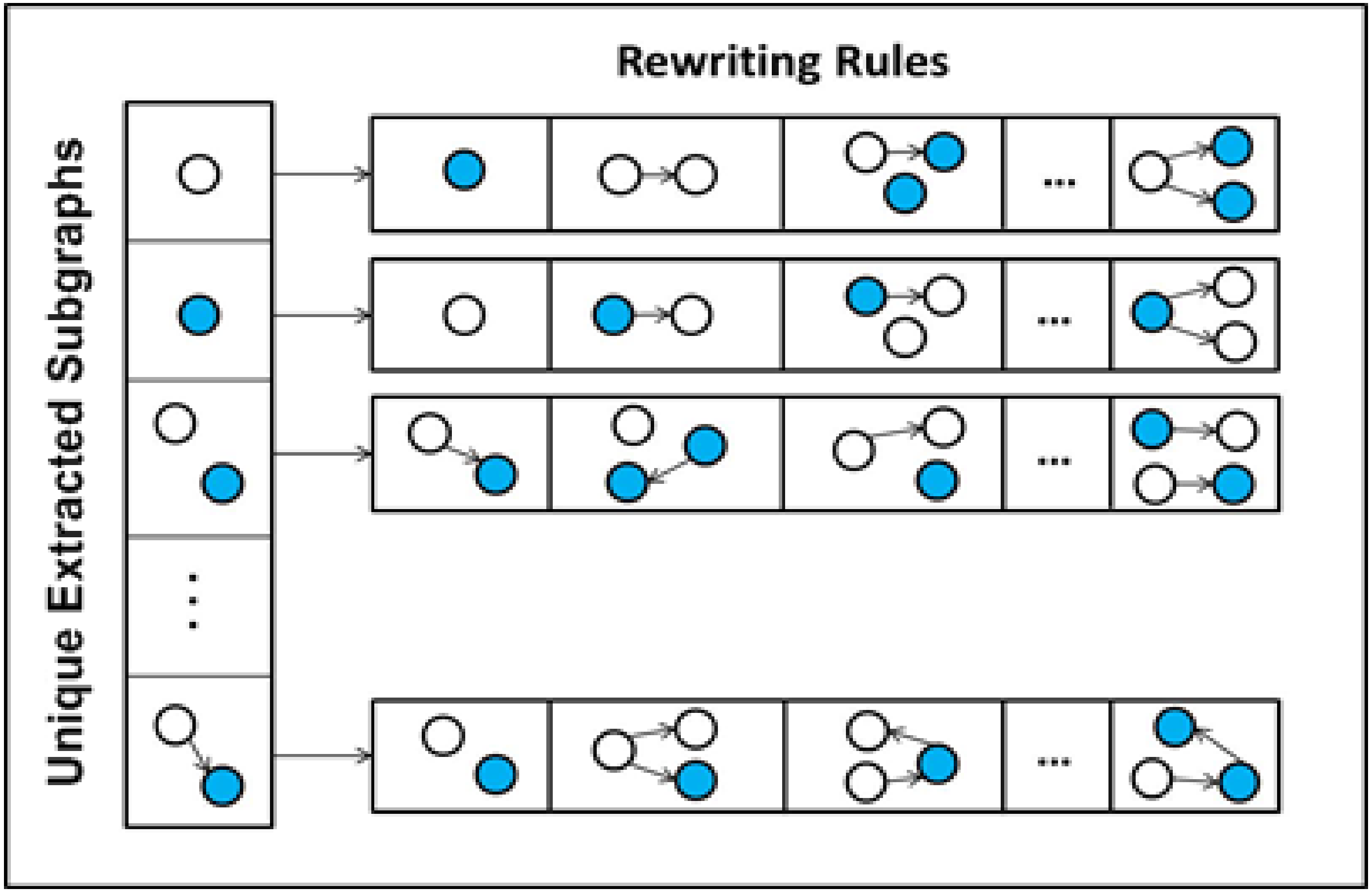
- Candidate models provided by user
  - Degree-based preferential selection
  - State-based preferential selection
  - Degree & State-based etc…
- Maximum likelihood method
  - Computes likelihood using each hypothetical model & accumulates log likelihood over time
  - Chooses the model with maximum likelihood

Algorithm

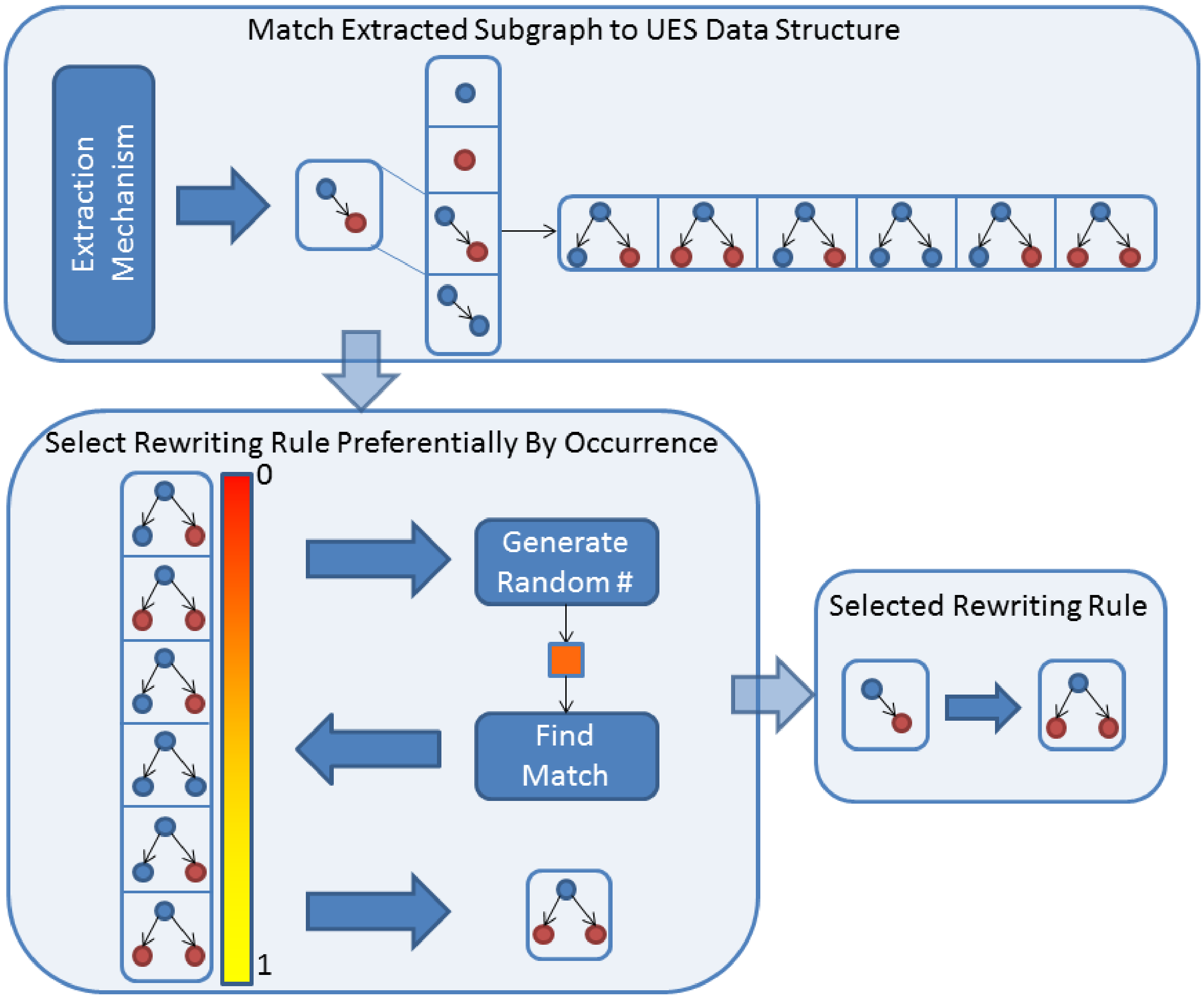


Input
Input Network
G^1
G^2
⋮
G^n
Extracted Subgraph
e_G^1
e_G^2
⋮
e_G^n
Prob. of Extraction Weight
P_{f1}
P_{f2}
P_{f3}
Likelihood Exponent
$\Sigma_t \ln P_{f1}$
$\Sigma_t \ln P_{f2}$
$\Sigma_t \ln P_{f3}$
$\hat{f} = f_3$
Less Likely
More Likely

Replacement mechanism identification: "What"



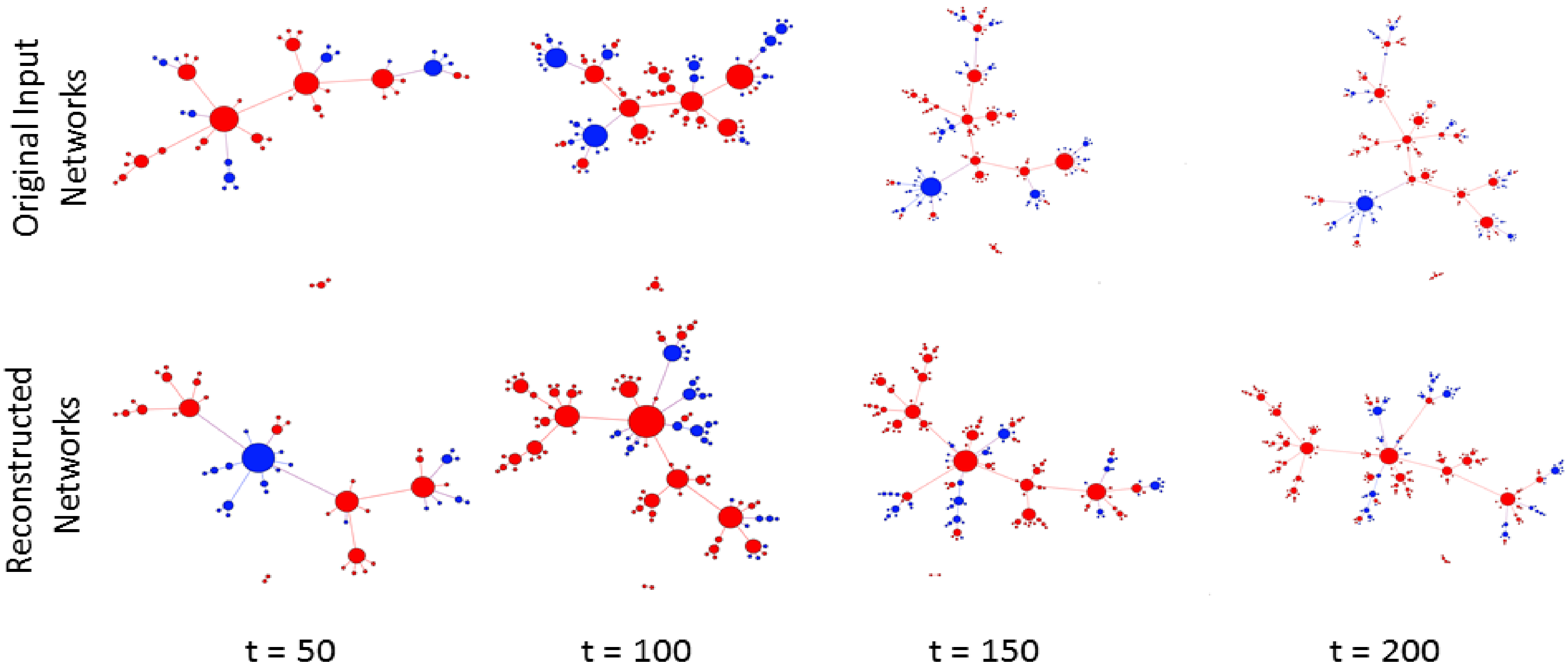
Algorithm

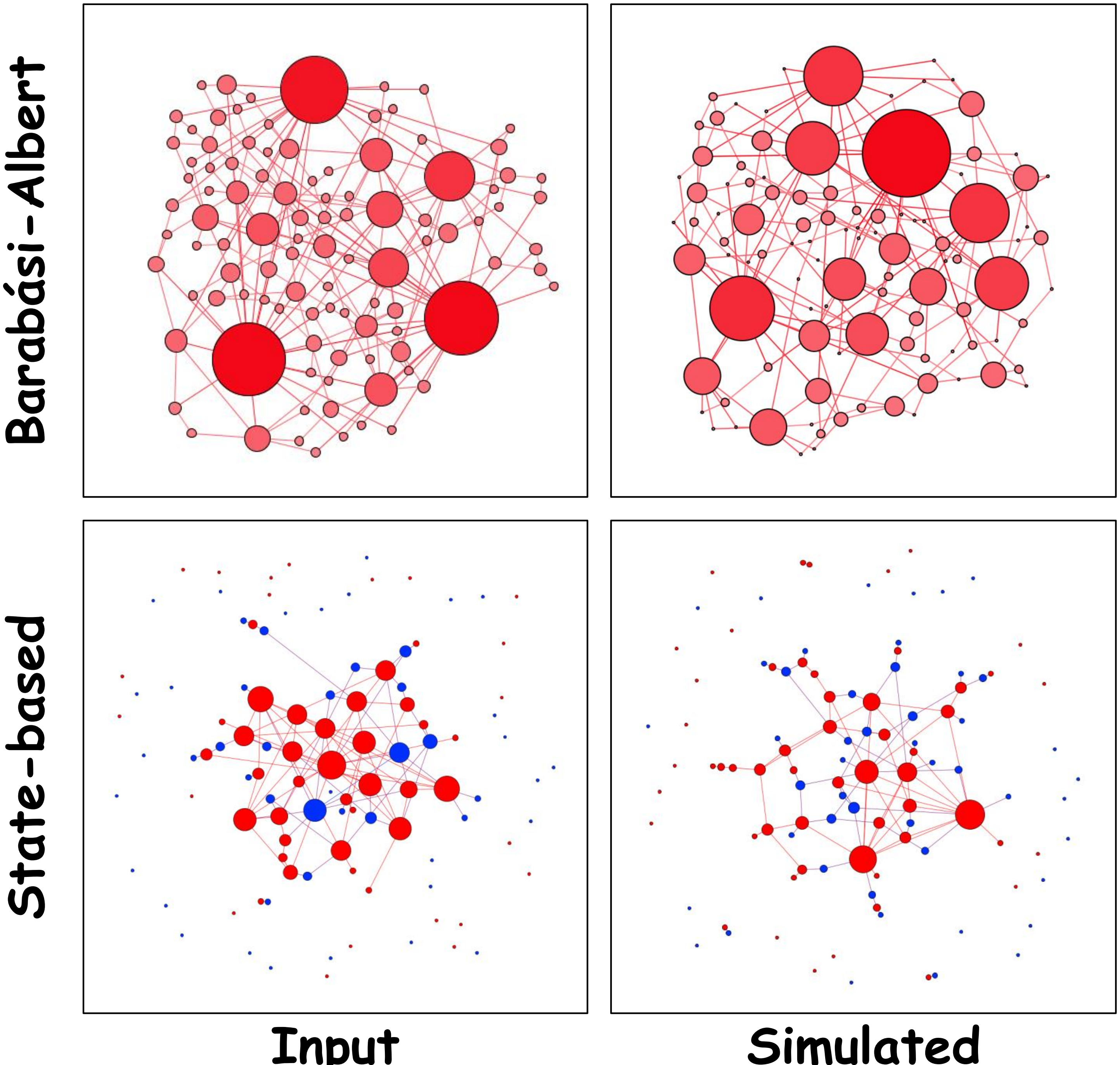


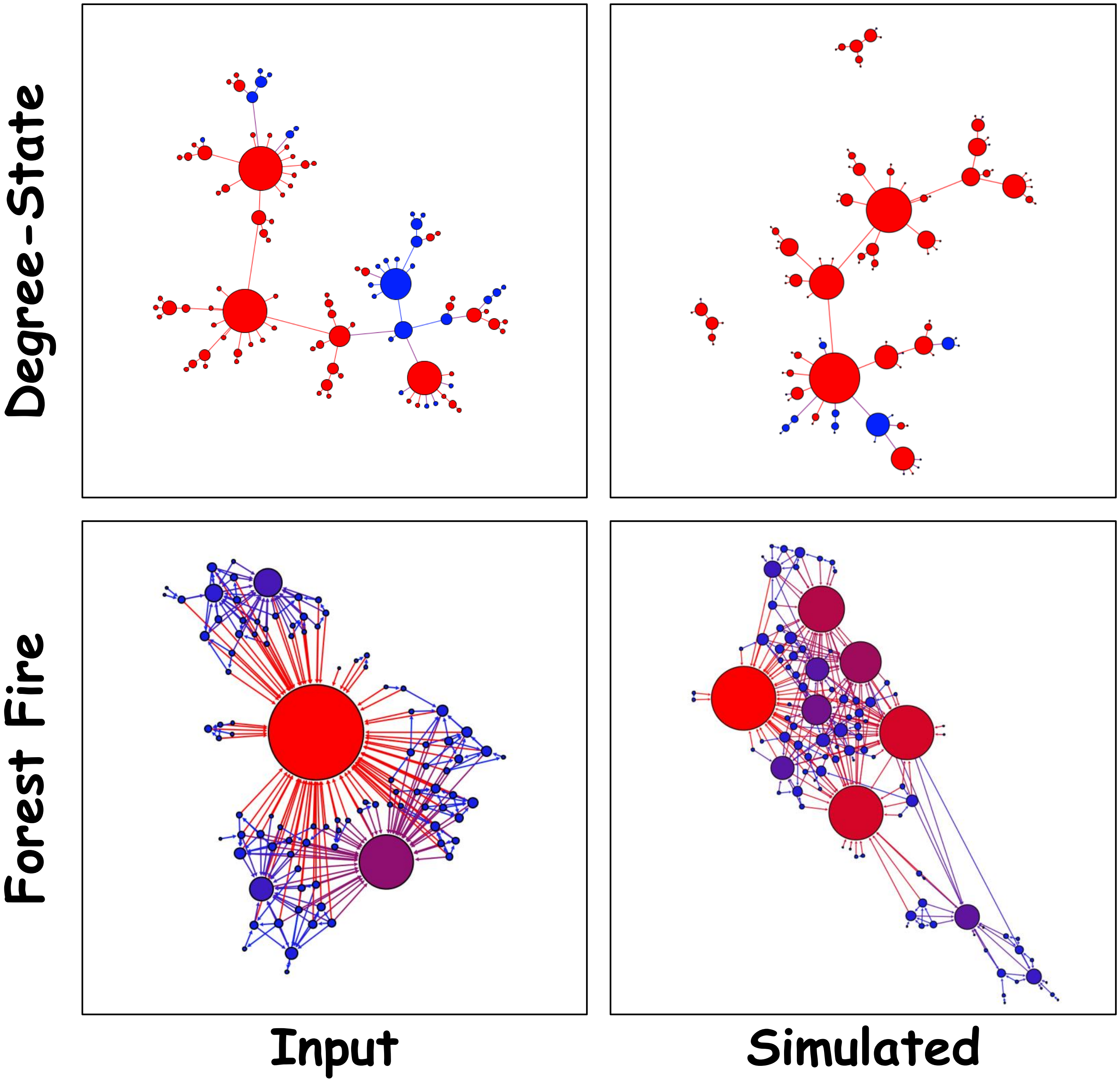
Match Extracted Subgraph to UES Data Structure
Extraction Mechanism
Select Rewriting Rule Preferentially By Occurrence
0
1
Generate Random #
Find Match
Selected Rewriting Rule

Results

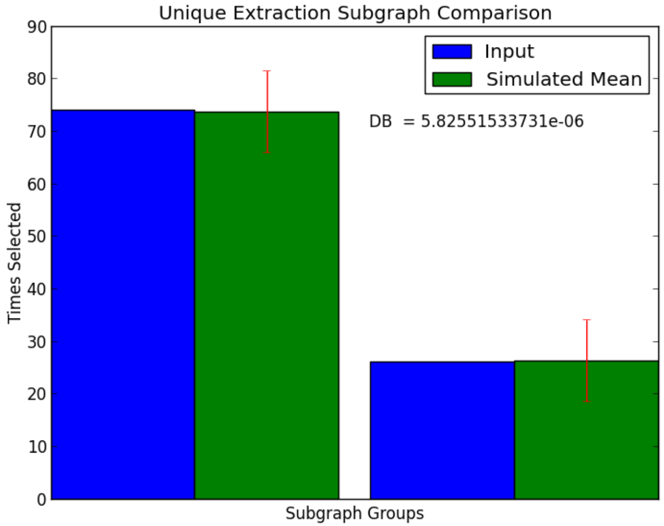
- Example: “Degree-state” networks



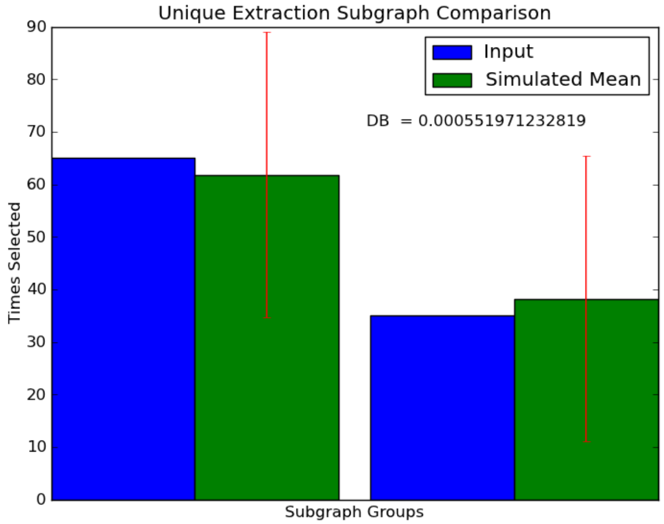

Barabási-Albert
State-based
Input
Simulated


Degree-State
Forest Fire
Input
Simulated

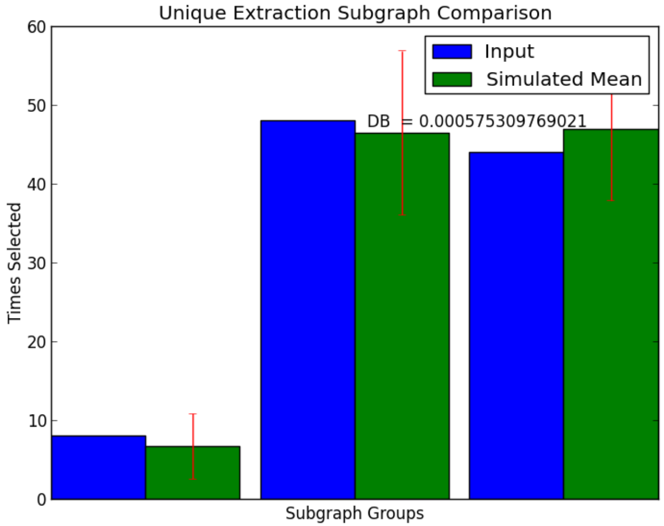
Barabási-Albert



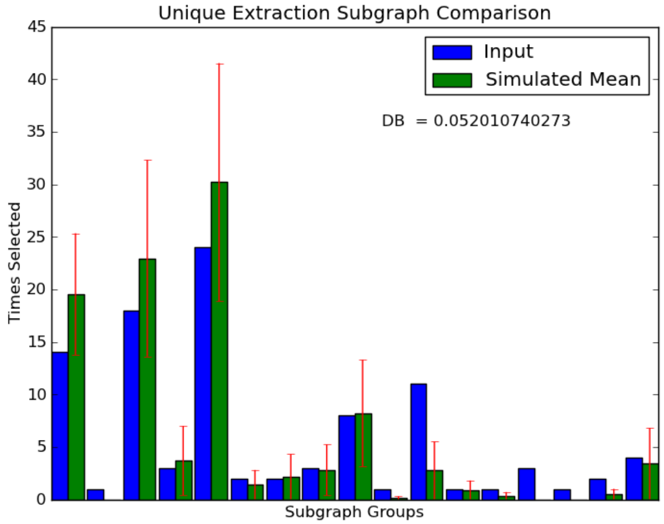
Degree-state



State-based



Forest Fire

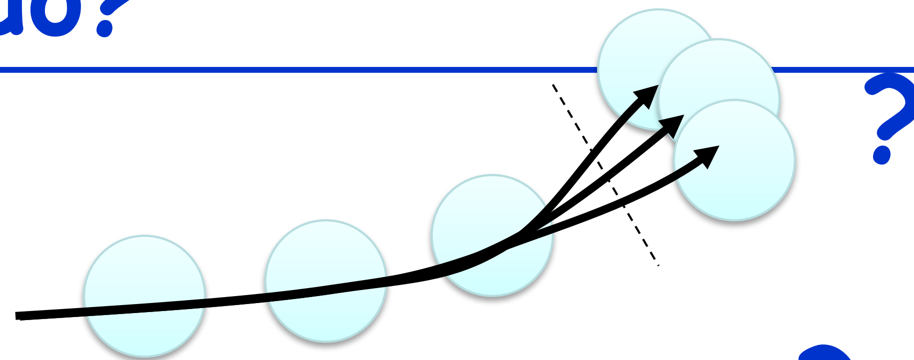


Comparison with other methods

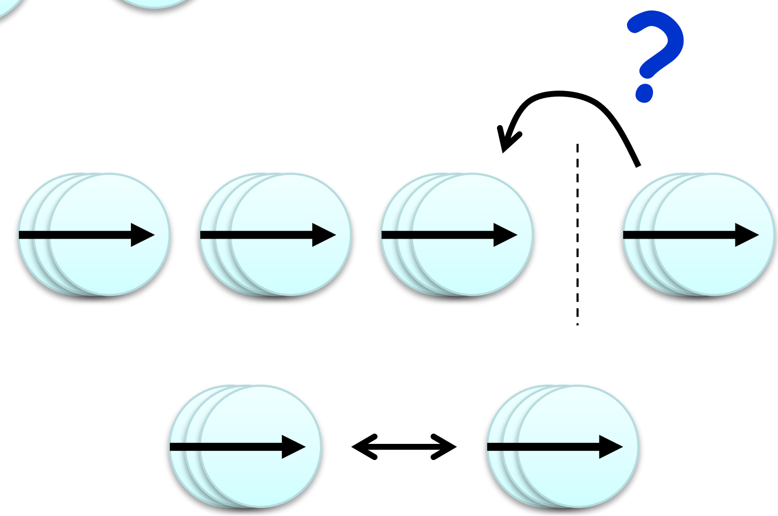
- PyGNA produces *generative models* using detailed state-topology information
  - Capable of *generative* simulation that is not available in statistical approaches (*e.g.*, Rossi et al. 2013)
- PyGNA models extraction and replacement as *explicit functions*
  - More efficient and flexible than graph-grammars (*e.g.*, Kurth et al. 2005)

What can we do?

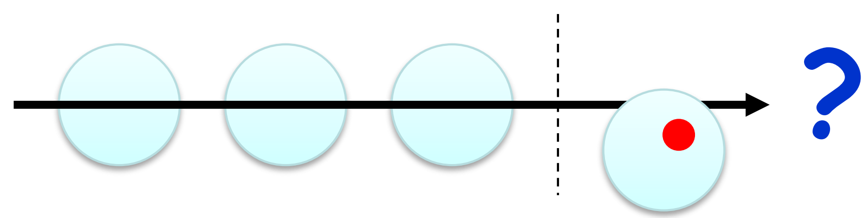
- Prediction



- Classification



- Anomaly detection



Summary

- State-topology coevolution of adaptive networks is a promising, unexplored area
  - Theory-driven approaches
    - Dynamical modeling, exhaustive rule search
    - Applications to social sciences etc.
  - Data-driven approaches
    - Application to operational network modeling
    - Automatic rule discovery from data

http://coco.binghamton.edu/NSF-CDI.html

Additional Topic:

Analysis of Adaptive Networks

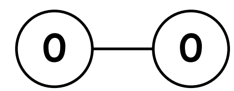
How to analyze adaptive network dynamics?

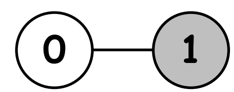
- Non-trivial coupling between network states and topologies are not easily handled in a simple analytical formula

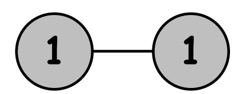
- But such couplings could be partially incorporated in analysis by considering densities of node "pairs"

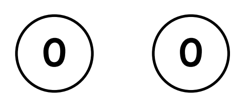
Pair approximation

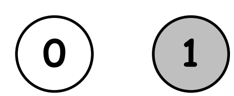
- Considers densities of every pair of nodes with states & connectivity (in addition to individual state densities)

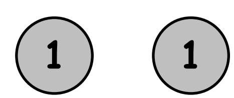
ρ_{00c} = density of 

ρ_{01c} = density of 

ρ_{11c} = density of 

ρ_{00n} = density of 

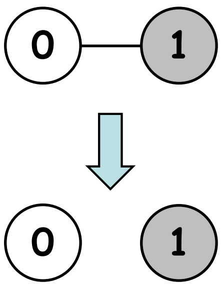
ρ_{01n} = density of 

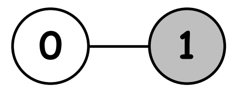
ρ_{11n} = density of 

Describes how these densities change over time

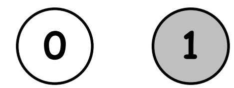
Example: Adaptive voter model

- Disconnect of a link:



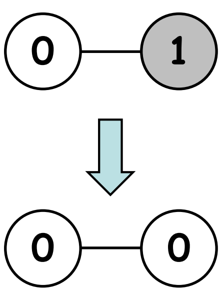


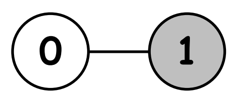
-1



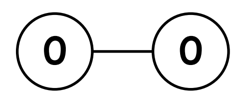
+1

- Change of an opinion:





-1?



+1?

(Any other densities affected too?)

Exercise

- Complete the number of changes in each pair density for the adaptive voter model on a random network
- Calculate how often each transition occurs
- Make a prediction using the pair-approximation-based model

Exercise

- Conduct pair approximation of the adaptive SIS model and study its dynamics

FYI: Moment closure

- Similar approximations are possible for densities of higher-order motifs
- Approximation techniques (including MFA, PA and higher-order ones) is called the "**moment closure method**"
  - Predicting the change of a "moment" (ρ_{00}) would require higher-order "moments" (ρ_{000}), but you "close" this open chain by assuming $\rho_{000} = \rho_{00}\,\rho_{00} / \rho_0$, etc.